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A systematic analysis of Non-Singleton Fuzzy Logic Systems: Control with Noisy Inputs

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ABSTRACT:- *A major asset of fuzzy logic systems is dealing with uncertainties arising in their various applications, thus it is important to make them achieve this task as effectively and comprehensively as possible. While singleton fuzzy logic systems provide some capacity to deal with such uncertainty aspects, non-singleton fuzzy logic systems (NSFLSs) have further enhanced this capacity, particularly in handling input uncertainties. This work broadly aims at removing the black boxed ness of Fuzzy Logic (Non-Singleton type) systems by plotting Fuzzy Basis Functions (FBF's) under different settings of membership functions of inputs, membership functions of antecedents and inference variants.*

In the initial part of the work, preliminaries define & clarify several issues & concepts connected with Non-Singleton FLS's and the reasons are bought out clearly as to why NSFLS's shall be superior to SFL's in case the Input is noisy. Several differences between SFL's & NSFL's are demonstrated through simulations under different inference schemes, a variety of membership functions of the input variables and the membership functions of antecedents.

KEYWORDS: *SFL, NSFL.*

INTRODUCTION

Fuzzy logic is an extension of Boolean logic by Lotfi Zadeh in 1965 based on the mathematical theory of fuzzy sets, which is a generalization of the classical set theory. By introducing the notion of degree in the verification of a condition, thus enabling a condition to be in a state other than true or false, fuzzy logic provides a very valuable flexibility for reasoning, which makes it possible to take into account inaccuracies and uncertainties. It provides a technique to deal with imprecision and information granularity. The fuzzy theory provides a mechanism for representing linguistic constructs.

Since we are all limited in our ability to perceive the world and to profoundly reason, we find ourselves everywhere confronted by uncertainty which is a result of lack of information (lexical impression, incompleteness), in particular, inaccuracy of measurements. The other limiting factor in our desire for precision is a natural language used for describing or sharing knowledge, communication, etc. We understand core meanings of word and are able to communicate accurately to an acceptable degree, but generally we cannot precisely agree among ourselves on the single word or terms of common sense meaning. In short, natural languages are vague.

It is important to observe that there is an intimate connection between Fuzziness and Complexity. As the complexity of a task (problem), or of a system for performing that task, exceeds a certain threshold, the system must necessarily become fuzzy in nature. Real world problems (situations) are too complex, and the complexity involves the degree of uncertainty – as uncertainty increases, so does the complexity of the problem. Traditional system modeling and analysis techniques are too precise for such problems (systems), and in order to make complexity less daunting we introduce appropriate simplifications, assumptions, etc. (i.e., degree of uncertainty or Fuzziness) to achieve a satisfactory compromise between the information we have and the amount of uncertainty we are willing to accept.

Based on both intuitive and expert knowledge, system parameters can be modeled as linguistic variables and their corresponding membership functions can be designed. Thus, nonlinear system with great complexity and uncertainty can be effectively controlled based on fuzzy rules without dealing with complex, uncertain, and error-prone mathematical models.

The variables in fuzzy logic system may have any value in between 0 and 1 and hence this type of logic system is able to address the values of the variables (called linguistic variables) those lie between completely truths and completely false. Each linguistic variable is described by a membership function which has a certain degree of membership at a particular instance. The human knowledge is incorporated in fuzzy rules. The fuzzy inference system formulates suitable rules and based on these rules the decisions are made. This whole process of decision making is mainly the combination of concepts of fuzzy set theory, fuzzy IF-THEN rules and fuzzy reasoning. The fuzzy inference system makes use of the IF-THEN statements and with the help of connectors present (such as OR and AND), necessary decision rules are constructed. The fuzzy rule base is the part responsible for storing all the rules of the system and hence it can also be called as the knowledge base of the fuzzy system. Fuzzy inference system is responsible for necessary decision making for producing a required output. The fuzzy control systems are rule-based systems in which a set of fuzzy rules represent a control decision mechanism for adjusting the effects of certain system stimuli. The rule base reflects the human expert knowledge, expressed as linguistic variables, while the membership functions represent expert interpretation of those variables.

II. OVERVIEW OF NON-SINGLETON FUZZY LOGIC SYSTEMS

The large variety of possible available information, together with the need for modeling all such information to determine a particular solution, necessitate the use of a very flexible information modeling technique. With this in mind, the formulation that provides the highest degree of latitude is a list of statements (rules) where each statement indicates the acceptability of a proposed solution based on some piece of information. The fuzzy formalism can provide a general framework to model certain or uncertain information in which an action is combined with a statement in an antecedent/consequent format and the individual statement solutions are aggregated to provide the overall solution. The set of statements comprise the fuzzy rule base, which is a vital part of a FLS The fuzzy inference engine combines the statements in the rule base according to approximate reasoning theory to produce a mapping from fuzzy sets in the input space to fuzzy sets in the output space. The fuzzifier maps crisp inputs to fuzzy sets defined on the input space and the defuzzifier maps the aggregated output fuzzy sets to a single crisp point in the output space. A fuzzy logic system processes crisp data at the input and produces crisp data at the output; therefore, a fuzzifier is used at the front of the system to convert crisp data to fuzzy data and a defuzzifier is used at the output of the system to convert fuzzy data into crisp data. The most widely used fuzzifier is the Singleton fuzzifier [3], [4], [9], mainly because of its simplicity and lower computational requirements; however, this kind of fuzzifier may not always be adequate, especially in cases where noise is present in the training data or in the data which is later processed by the system. A different approach is necessary to account for uncertainty in the data, which is why we direct our attention at NSFLS's.

NSFLS's are a family of systems that have non-Singleton fuzzy sets as inputs. The structure of the rulebase is identical as in the Singleton FLS case, except that input linguistic variables are allowed to take set values (instead of single-point values). NSFLS's are a powerful generalization of Singleton FLS's and provide a mathematically tractable method to treat input uncertainty. Non-Singleton fuzzifiers have been used successfully in a variety of applications [1], [2], [5], [6], [7], [8].

Non-Singleton input has also been used in turning process automation [7] to represent a human operator's actions in the fuzzy rule base, in the design of fuzzy control algorithms [6], and in fuzzy information and decision-making [8]. NSFLS's have non-singleton fuzzy sets as input; however the rule base is identical to singleton fuzzy systems, except that I/P linguistic variables take set values instead of single-point values. NSFLS's are a powerful generalization of SFLS's providing a mathematical method to treat the I/P uncertainty in SFLS's.

A non-singleton fuzzifier implies that an I/P value x, that is crisp is the most likely correct value among all values in its immediate neighborhood, but since the I/P is noisy hence the neighboring values would be correct but surely to a lesser degree and thus the quantity of noise present in the I/P governs the shape of the I/P membership function although it would be symmetric about$x$. Since signals could be corrupted by noise hence, adjacent pointsmay also be correct values, but to a lesser degree. In a more formal way, we can define non-singleton fuzzification as a non-singleton fuzzifier for which $\mu_X(x_i) = 1$, for $x_{i} = x$ and $\mu_X(x_i) = 0$, for $x_i \neq x$. μ_X decreases from unity as we move away from $x_{i} = x$. Here where μ_X is the membership function of the fuzzy set X &x being the elements of UOD.

RELATED WORK

This section deals with the general results of an NSFLS.A fuzzy rule with q consequents can be decomposed into q rules, each having the same antecedents & one different consequent.

Let there be M rules with p antecedents & only one consequent in the rule base.

Let the r-th rule be R^r& its general form would be:

$$R^r\text{: IF } u_1 \text{ is } A^r_1 \text{ AND } u_2 \text{ is } A^r_2 \text{ AND...AND } u_p \text{ is } A^r_p \text{ THEN } v \text{ is } C^r$$

Whereu_k, $(k=1,...,p)$ is the I/P linguistic variable & v the O/P linguistic variable.A^r_1&$C^r$are subsets of different UOD's.

Each rule here can be viewed as a fuzzy relationR^r from a set$U$ to V, where Uitself is the Cartesian product :

$$U= U_1 \, x \ldots \, xU_p$$

$R^r$is further a sub set of the Cartesian product:

$$U \, x \, V = \{(x, y): x \in U, y \in V\} \text{ with} x \equiv (x_1, x_2, \ldots, x_n)$$

A continuous & a discrete case as per systems formulation may be defined as:

$$R^r \equiv \int_{U \, x \, V} \mu_{R\,r}(x, y)/(x, y)$$

$R^r$is a multivariate membership function $\mu_{R\,r}(x, y)$

Transforming $\mu_{R\,r}(x, y)$ relationship into t-norms, we get

$$R^r \equiv \int_{U1} \ldots \int_{U\,p} \int_V \mu_{A1}{}^r(x_1) *_{\ldots} * \mu_{Ap}{}^r(x_p) * \mu_C{}^r(y) / (x, y)$$

where$*$denotes t-norm&$\int$denotes the union of individual points, $x_i$of each set in continuum & '/' denotes a tuple.

Let the I/P to R^r be denoted by I where I is a subset of a p-dimensional Cartesian product space given by:

$$I \equiv \int_{U1} \ldots \int_{U\,p} \mu_{X1}(x_1) *_{\ldots\ldots} * \mu_{Xp}(x_p) / (x)$$

Here $X_k \subset U_k (k = 1, \ldots, p)$ are fuzzy sets describing the I/P's.

Each I/P fuzzy set X_k has non zero membership value only at a single point reducing I to a set with single point $x_k \in U$ where as in a non-singleton case it is a not a single point because it has several values.

Using the compositional rule of Inference, the fuzzy subset Y^r of V induced by $I \in U$ is obtained by composition of I and R^r as:

$$Y^r = I \circ R^r = \sup_{x \in U} (I * R^r).$$

Since all unions in R^r as well as in I, over U_k, $(k = 1, \ldots, p)$ are over the same spaces, therefore, Y^r can be written as:

$$Y^r = [\sup_{x \in U} (\int_{U1} \ldots \int_{Up} \int_V \mu_{X1}(x_1) * \ldots * \mu_{Xp}(x_p) * \mu_{A1}{}^r(x_1) * \ldots * \mu_{Ap}{}^r(x_p) * \mu_c{}^r(y)/(x))]/(y)$$

Now since the supremum is only over $x \in U$, so by the commutativity & monotonicity properties of t-norms, Y^r can be written as:

$$Y^r = \int_V \mu_c{}^r(y) * [\sup_{x \in U} (\int_{U1} \ldots \int_{Up} \mu_{X1}(x_1) * \ldots * \mu_{Xp}(x_p) * \mu_{A1}{}^r(x_1) * \ldots * \mu_{Ap}{}^r(x_p)/(x))]/(y)$$

Every t-norm in the above equation is taken to be operating on a pair of membership functions. The computation of t-norms over all the points in the corresponding spaces of the two membership functions in the same space can be written as:

$$Y^r = \int_V \mu_c{}^r(y) * [\sup_{x \in U} (\int_{U1} \ldots \int_{Up} [\mu_{X1}(x_1) * \mu_{A1}{}^r(x_1)] * \ldots * [\mu_{Xp} * \ldots \mu_{Ap}{}^r(x_p)] / (x))]/(y)$$

The supremum above is over all points x in U. By monotonicity property of t-norms, this supremum is attained when each term in square bracket attains its supremum. Therefore, let kth bracketed expression be denoted by:

$$\mu_{Qk}{}^r(x_k) \equiv [\mu_{Xk}(x_k) * \mu_{Ak}{}^r(x_k)]. \text{Where } \mu_{Qk}, \mu_{Xk}, \mu_A{}^r \in [0, 1]$$

Assume that $\mu_{Qk}{}^r$ is a function whose supremum can be evaluated & let $x^r_{k,sup}$ denote the point in UOD U_k where supremum is attained.

Hence fuzzy subset Y^r of V induced by $I \in U$ could be denoted as:

$$Y^r = \int_V \mu_c{}^r(y) * T^p_{k=1} \mu_{Qk}{}^r(x^r_{k,sup}) /(y)$$

Where $T^p_{k=1}$ denotes p-1 t-norm operations.

For NSFLS's, each U_i, $(i = 1, \ldots, p)$ and V are countable/finite & the ith antecedent membership function $\mu_{Ai}{}^r$ and the consequent membership function $\mu_c{}^r$ have non-zero membership values at discrete points $x_{i,1}, \ldots x_{i,n_i}$ and $y_1, \ldots, y_m$ respectively, hence the rth rule R^r is given by:

$$R^r = \sum_{i_1=1} \ldots \sum_{i_p=1} \sum_{j=1} \mu_{A1}r(x_{1}, i_1) * \ldots * \mu_{Ap}r(x_p, i_p) * \mu_C r(y_j) /(x_{1,i1}\ x_{2,i2}, \ldots, x_{p,ip}, y_j)$$

Here Σ denotes the union of individual points of each set. The p-dimentional input to R^r is given by:

$$I = \sum_{i_1=1} \ldots \sum_{i_p=1} \mu_{A1}{}^r(x_1, i_1) * \ldots * \mu_{xp}(x_p, i_p) / (x)$$

where $x = (x_1, i_1, x_2, i_2, \ldots, x_p, i_p)$

Now using the Compositional Rule of Inference & the commutativity & monotonicity properties of a t-norm, we can write the discrete output fuzzy set as:

$$Y^r = \sum_{j=1} \mu_C{}^r(y_j) * \sup_{x \in U} (\sum_{i_1=1} \ldots \sum_{i_p=1} \mu_{X1}(x_{1,i1}) * \ldots * \mu_{Xp}(x_{p,ip}) * \mu_{A1}{}^r(x_{1,i1}) * \ldots * \mu_{Ap}{}^r(x_{p,ip}) /(x))/(y_j)$$

Since each UOD U_k is finite, supremum is the same as the maximum, therefore, the maximum needs to be calculated over all $x \in U$

The parenthetical term in the above equation will be maximized when every term is maximized.

$$\mu_{Qk} r(x_{k,ik}) \equiv [\mu_{Xk}(x_{k,ik}) * \mu_{Ak} r(x_{k,ik})]$$

If the global maximum $x^r_{k,max}$ of each $\mu_{Qk} r(x_{k,ik})$ can be found then

$$Y^r = \sum_{j=1}^{m} \mu_C r(y_j) * T^p_{k=1} \mu_{Qk} r(x^r_{k,max}) / (y_j)$$

Here Y^r is discrete.

As the NSFLS has M rules, the final O/P fuzzy set Y is obtained by t-conorm aggregation of the individual fuzzy set O/P of each rule as:

$$Y = Y^1 + Y^2 + \ldots + Y^M$$

Where + denotes t-conorm (s-norm).

RESULTS AND CONCLUSION

In presenting the differences between the non-singleton & singleton FLS, we consider the case of a one-rule Single Input Single Output FLS with Gaussian membership function for the antecedent &a Product Inference. For NSFLS case, the I/P is also fuzzified using a Gaussian membership function for the I/P.

Fig. 3.1 shows O/P fuzzy set for singleton case of one rule SISO with $\sigma_A = 10$ (standard deviation of antecedent) $m_A = 45$ (mean of antecedent), I/P $x_1' = 25$ (singleton Input), $\sigma_C = 15$ (standard deviation of consequent), $m_C = 55$ (mean of consequent), with Gaussian antecedent & consequent membership functions.

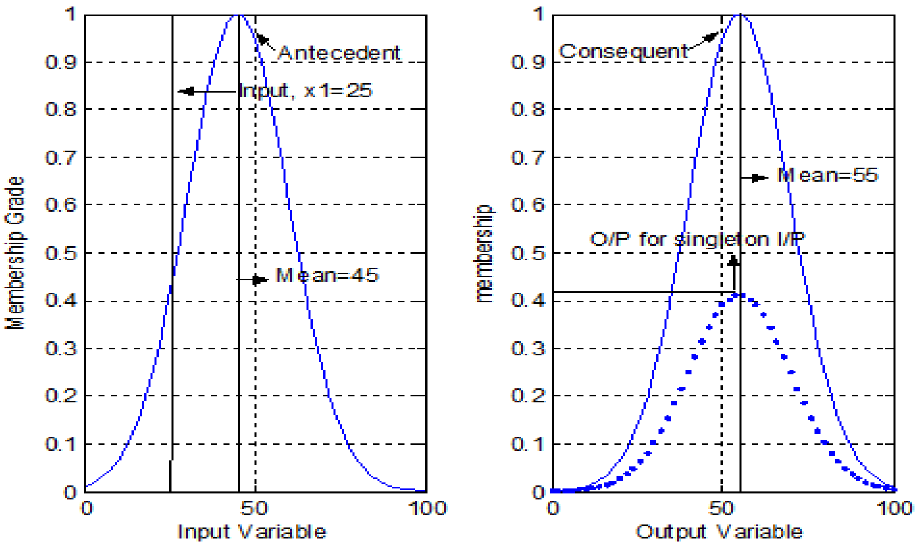

Figure 1:Composition of Rule with Singleton Input & Corresponding Output Fuzzy Set for SISO FLS.

In Fig.2 similar results, as in Fig 1 are depicted, but with I/P also fuzzified using a Gaussian membership function, with I/P $m_{X_1} = x_1' = 25$, standard deviation of I/P $\sigma_{X_1} = 7$ & hence we observe the Gaussian nature of the product function & location of its maximum value at
$x = x^1_{1,max} = 30$.

In this case the O/P membership function for NSFLS is of greater maximum height than that of the SFLS, as can be seen by comparing Fig. 1 & Fig. 2 & this is true for all shapes of membership functions. We also notice that in this case only one I/P value over the entire UOD of the antecedent, the Scaling Factor Difference (sfd) is only a single value &, therefore, denoted by a single point if drawn corresponding to single I/P value. This value varies with the mean of I/P & the standard deviation of the I/P.

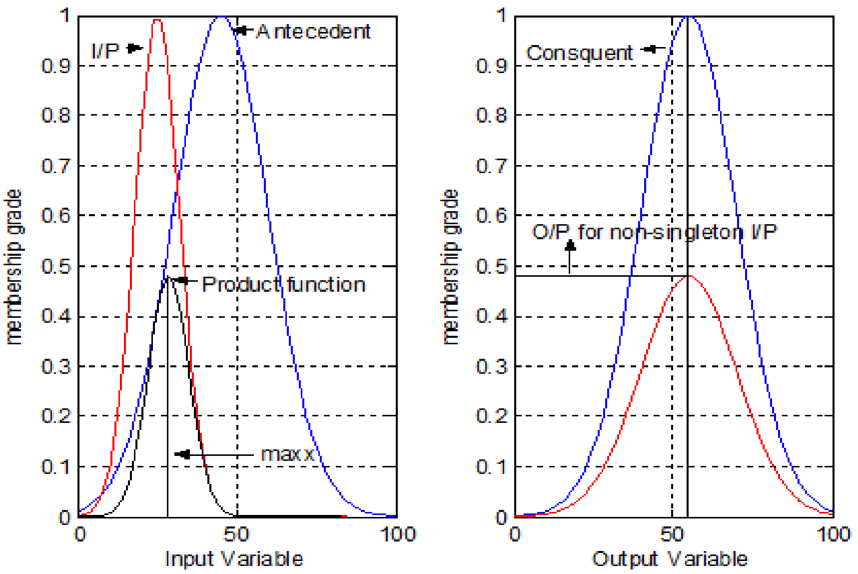

Figure 2:Composition of Rule with Non-Singleton Input & Corresponding Output Fuzzy Set forSISO NSFLS.

Figure 3 and 4 show the comparison of Control Performance for NSFLS & SFLS.

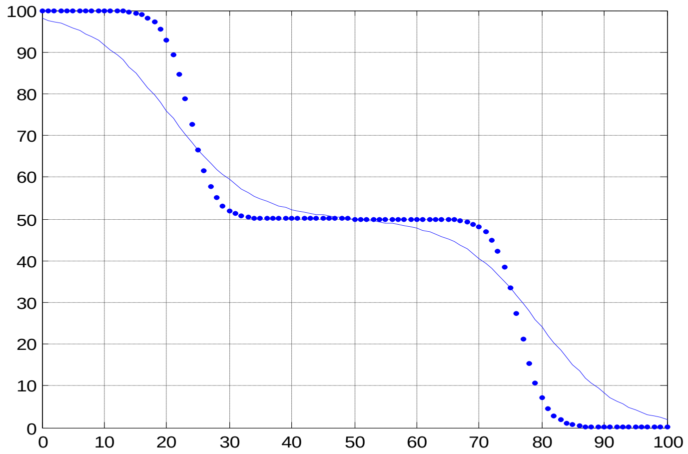

Fig. 3: Control Curve for Gaussian Inputs & Antecedents – Product Inference.

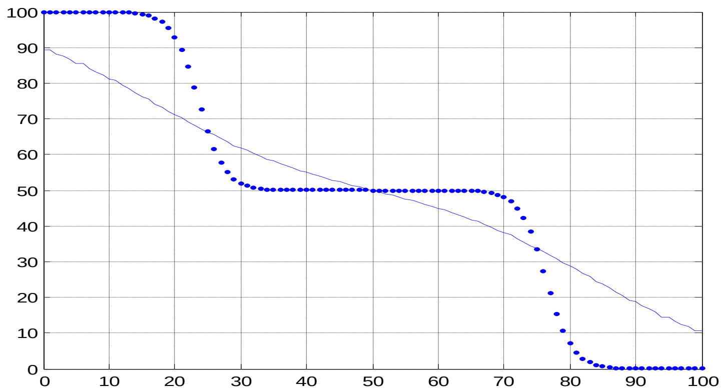

Fig. 4: Control Curve for Gaussian I/P & Antecedents – Min Inference.

When we have one variable , four rules & the Inputs & antecedents are Gaussian, the Control Curves for Product & Min inference are shown in Fig. 3 & Fig.4 respectively with dotted (**..**) curve representing SFLs & the lined (__) curve representing NSFLS.

Non-Singleton fuzzification is especially useful in caseswhere the available training data or the input data to the fuzzy logic system are corrupted by noise. Conceptually, the non-Singleton fuzzifier implies that the given input value is the most likely value to be the correct one from all the values in its immediate neighborhood; however, because the input is corrupted by noise, neighboring points are also likely to be the correct values, but to a lesser degree.

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