

**COST OPTIMIZATION OF RIGIDLY JOINTED REINFORCED CONCRETE  
FRAME STRUCTURES**Waqar Ahmad<sup>1</sup>, Bashir Alam<sup>1</sup><sup>1</sup>Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

**Abstract** —This work aims at developing an efficient, generalized, automated and computer based solution for multi-dimensional optimization problem. An automated procedure is developed for optimized design of reinforced concrete frame structures. The objective function includes the total cost of the structure i.e. the sum of the cost of concrete, steel and formwork of all the elements of the structure. The design variables include the member sizes and the member reinforcement for both beams and columns. The constraints arise from the strength and serviceability requirements of American Concrete Institute (ACI) specifications and codes, based on strength design method are all satisfied. A Computer program is developed by interlinking Visual Basics .NET (VB.Net) programming language and MATLAB to automate the performance of the structural analysis, design and optimization of the frame structures. An algorithm is developed for finding out the local optimum of each element by creating a complete design profile of the element. The global optimum is thus obtained by summing the individual optima. Two examples are solved to observe the behavior of the developed algorithm.

**Keywords-** Optimization, Reinforced Concrete Buildings, Design.

**I. INTRODUCTION**

In the conventional design process, the capacity of the structure is checked against the demand by trial and error. Once the capacity exceeds the demand the designer usually terminates the iteration process. The design thus obtained is feasible but not necessarily optimized. The basic aim of a structural optimization problem is to find such values of member sizes and steel reinforcement which fulfill the demand at minimum cost. In other words to select an optimal set of design variables at which the pre-defined objective function can be minimized meeting all the constraints. The objective function is a criterion for deciding the best design among a pool of various feasible designs. It can be cost of materials used in the structure, weight of the structural components or combination of these or similar factors. The constraints are the restrictions set by the design specification and codes on design variables. A number of optimization techniques are developed for various structural optimization problems. Kaveh and Abbasgholizadeh (2011) worked on optimization of steel sway frames using an optimization approach called Big Bang and Big crunch theory. Andjelic and Milo (2012) worked on the optimization of thin-walled I-beam subjected to stress constraint. The I-section was considered as the objective of the optimization as it is widely used steel section. Rizwan et al. (2012) worked on the optimization of combined footing. They found Box method an efficient approach for economic design of footing. Alam and Haque (2012) worked on design optimization of planar frames with rigid joints using the modified complex approach. Masami et.al. (2013) worked on a new parameter-free shape optimization method for the optimized free-form design of space frame structures. Large-scale shape optimization problems can also be solved using their proposed method. Beghini et.al. (2014) worked on developing a relationship between architecture field and engineering field by focusing structural and geometrical optimization. Khalid et al. (2014) successfully optimized pre-stressed girders using the modified complex method of Box. Kaveh and Khayatazad (2014) worked on the optimization of retaining wall using the new meta-heuristic approach called Ray optimization method. Kapgate et.al. (2015) worked on optimal design of semi rigid jointed frame structures. They found that the weight minimization is a function of various factors such as choice of design parameters, restrictions, planning of objective function and other related factors. Preethi and Arulraj (2016) worked on the optimization of reinforced concrete column using MATLAB software. Gdu et.al. (2016) worked on the application of Artificial Bee Colony (ABC) algorithm. They found that the algorithm faces difficulty in solving problems which are not linear, in other words linear programming problems can easily be solved using the said approach. Ukritchon and Keawsawasvong (2016) worked on development of a practical method for the optimal design of continuous footing, using ant-colony optimization.

The basic difference in the techniques lies in the development of the design space in which the optimal design is searched. For an accurate search the accuracy and broadness of the said design space is of key importance.

**II. Optimum Design Problem**

The optimum ultimate strength design problem of a rigidly jointed plane frame has the following contributing items.

**A) Design Variables**

The design variables for the current cost optimization problem are listed below:

- 1) Depth of beam ( $b_d$ )
- 2) Width of beam ( $b_w$ )
- 3) Depth of column ( $h$ )
- 4) Width of column ( $b$ )
- 5) Reinforcement area in beam
- 6) Reinforcement area in column

**B) Objective Function**

The objective function is the main controlling factor of any optimization problem. In current problem, the total cost of the frame structure is considered as the objective function for the analysis to be carried out. This includes the total cost of concrete including formwork and the total cost of reinforcement.

$$F = \text{Total Cost} = C_{\text{concrete}} + C_{\text{reinforcement}}$$

$$F = (V_{\text{concrete}} \times R_{\text{concrete}}) + (W_{\text{reinforcement}} \times R_{\text{reinforcement}})$$

Where,

F = Total cost of the structure

$C_{\text{concrete}}$  = Total cost of concrete including formwork

$C_{\text{reinforcement}}$  = Total cost of reinforcement

$V_{\text{concrete}}$  = Total volume of concrete

$R_{\text{concrete}}$  = Rate of concrete and formwork per unit volume

$W_{\text{reinforcement}}$  = Total weight of steel reinforcement

$R_{\text{reinforcement}}$  = Rate of steel reinforcement per unit weight

### C) Design Constraints

To satisfy the safety and serviceability requirements some restrictions are imposed on the structure which are termed as Design Constraints. Design constraints are mainly the restrictions imposed by the design codes which keep some parameter of the design to lie within specified limits. These restrictions can be explicit such as the restrictions on dimensions of the elements, or implicit such as the restrictions on stresses and displacements.

In current problem the design limitations set by the American Concrete Institute (ACI) specifications and codes [14] for concrete structures are considered as design constraints and are all satisfied.

### III. Problem Description

The optimum ultimate strength design problem of a rigidly jointed frame can be stated as “Find values of the design variables which minimize the objective function satisfying all the design constraints”. Such a set of design variable can be called the personal best of each element. In case of rigidly jointed frame, the personal best of each element is also affected by the personal bests of the neighboring connected elements. Thus in this case the optimization problem is a multi-dimensional problem. Each element must find its personal best taking into the account the personal bests of its neighboring connected elements.

The problem can be formulated as a mathematical programming problem as follows.

Find the beam and column design vectors

$$B = B_i = [(b_d)_m, (b_w)_m] \quad (1)$$

$$(i = 1, 2, \dots, J), (m = 1, 2, \dots, M)$$

$$C = C_i = [(h)_n, (b)_n] \quad (2)$$

$$(i = 1, 2, \dots, J), (n = 1, 2, \dots, N)$$

So that the objective function,

$$F = \sum (V_{\text{concrete}} \times R_{\text{concrete}}) + (W_{\text{reinforcement}} \times R_{\text{reinforcement}})$$

attains a minimum value among all the feasible designs.

Where  $B_i$  and  $C_i$  are the elements of beam design vector and column design vector respectively, M and N are the numbers of beams and columns respectively and J is the number of indices of design space.

### IV. Solution Procedure

An algorithm is developed to integrate the following two modules

A) Module for checking the design constraints

B) Module for Creating Design profile

#### A) Module for checking the design constraints

This module decides the feasibility of a design on the basis of provisions set by the American Concrete Institute for concrete frame structures. These provisions include maximum and minimum reinforcement requirements, beam/column capacity ratios, limits on torsion and axial stresses as well as the allowable dimensions for the beams and columns. This module strictly applies the strength constraints whereas the serviceability constraints are kept optional. The module is integrated with the second module for generating feasible designs.

#### B) Module for Creating Design profile

The term design profile refers to the complete history of an element’s objective function for all the feasible design vector indices unless the termination criteria is reached. This module creates a complete design profile of each element of the structure. This module consist of two phases i.e. the analysis phase and the design phase.

In the structural analysis phase, the analysis is performed using the direct stiffness approach. Information pertaining to the structure is assembled and recorded. A transformed joint stiffness matrix is obtained by the contributions from all the individual transformed member stiffness matrices. The joint displacements, reactions and member end actions are computed for all members.

In the design phase, the all the members are designed for the applied loads as well as for the self-weight. In the design of concrete beams, areas of steel for flexure are calculated based on the beam moments, load combination factors and other criteria set by American Concrete Institute. All beams are designed for major direction flexure only. The beam section is designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. The beam is first designed as a singly reinforced beam. If the beam section is not adequate, the required compression reinforcement is calculated. Similarly the columns are designed for axial force as well as the uniaxial moments.

The beams are designed and optimized prior to the columns. The proposed optimization process can be mathematically summarized as:

Assign equal cross-sectional areas to all column members and for each beam element, find the initial beam objective function vector

$$(F_{\text{initial}})_m = F_m = [(b_d)_m, (b_w)_m, (A_s)_m] \quad (1)$$

In which,  $b_w = r, b_d = r, r+1, \dots, 2r$ ,  $m = 1, 2, \dots, M$

Where,  $r \geq 4$ . The final value of  $r$  depends on the termination criteria.

Now, find the initial beam optimal design vector, which are the values of design variables corresponding to the minimum value of the initial objective function vector

$$B_{\text{initial}} = B_i = [(b_d)_i, (b_w)_i]_{\text{Min } (F_{\text{initial}})} \quad (2)$$

In which  $i = 1, 2, \dots, J$

Assign the values of initial beam design vector to the corresponding beam members and for each column element, find the initial column objective function vector

$$(F_{\text{initial}})_m = F_m = [(h)_m, (b)_m, (A_s)_m] \quad (3)$$

In which  $b = r, h = r, r+1, \dots, 3r$ ,  $m = 1, 2, \dots, N$

Where,  $r \geq 4$ . The final value of  $r$  depends on the termination criteria.

Now, find the initial column optimal design vector, which are the values of design variables corresponding to the minimum value of the initial objective function vector

$$C_{\text{initial}} = C_i = [(h)_i, (b)_i] \quad (4)$$

In which  $i = 1, 2, \dots, J$

Assign the values to the corresponding column members. Recalculate the optimal beam design vector from the modified objective function vector followed by the calculation of optimal column design vector until both the vectors converges to individual constant values.

Where  $F_m$  is the objective function vector of member  $m$ ,  $b_d$  is the beam depth,  $b_w$  is the beam width,  $A_s$  is the area of steel in member  $m$ ,  $M$  and  $N$  are the number of beam and column members respectively,  $B_i$  and  $C_i$  are the elements of beam design vector and column design vector respectively,  $b$  is the column section dimension along major axis-1,  $h$  is the column section dimension along major axis-2.  $J$  is the number of indices of design space.

The value of  $r$  in eq. (1) and eq. (3) is arranged such that it results in a number of sequences of the objective function in the design profile. The termination criteria confirms the increasing trend of the objective function vector by searching the minimum value of objective function in successive sequences. This is mathematically described in eq. (5)

$$(S_{\text{min}})_q / (S_{\text{min}})_w \leq 1 \quad (5)$$

In which  $q = 1, 2, 3, 4, \dots, 50$  and  $w = q, q+1, \dots, z$

Where  $(S_{\text{min}})_q$  is the minimum value of objective function in a sequence  $q$ ,  $(S_{\text{min}})_w$  is the minimum value of objective function in a  $z$  successive sequences starting from sequence  $q$ .

## V. Examples

Two numerical examples are solved with different loading combinations and number of storeys to demonstrate the feasibility of the proposed automated method. The results of the examples are generated with a Visual Basic.Net computer program linked with MATLAB, executed on a Core i3, 2.4 GHz laptop computer with 2 GB of RAM. The program is completely automated and it attempts to integrate both the modules discussed above to obtain the optimized design of any plane concrete frame structure.

### Example 1: Three-Bay Two-Storey Plane Frame

A three-bay, two-storey concrete rigid joint frame shown in Figure 1, is optimized to test the performance of the developed algorithm. For this problem the cost of steel is taken as 1, 20,000 Rs/Ton whereas the cost of concrete is taken as 260 Rs/Cft. The concrete compressive strength is taken as 4000psi whereas the yield strength of steel is taken as 60,000 psi. The design optimization problem is to find the cross-sectional dimensions of all the members such as to minimize the cost of the concrete frame structure. An upper limit of 50in is imposed on all the design variables. The length of each beam member is 24ft whereas the height of each column member is 12ft. The skeletal geometry of the structure, load type and magnitudes for this problem are shown in Figure 1. The beam members of storey 1 are subjected to uniformly distributed loading of magnitude 1.5 kips/ft. whereas the beam members of storey 2 are subjected to uniformly distributed loading of magnitude 1.0 kips/ft. Both these loadings are live loads whereas the self-weight of the members is taken as dead load. The following load combinations are used for this example:

1.4D (ACI 9-1)  
 1.2D + 1.6L (ACI 9-2)

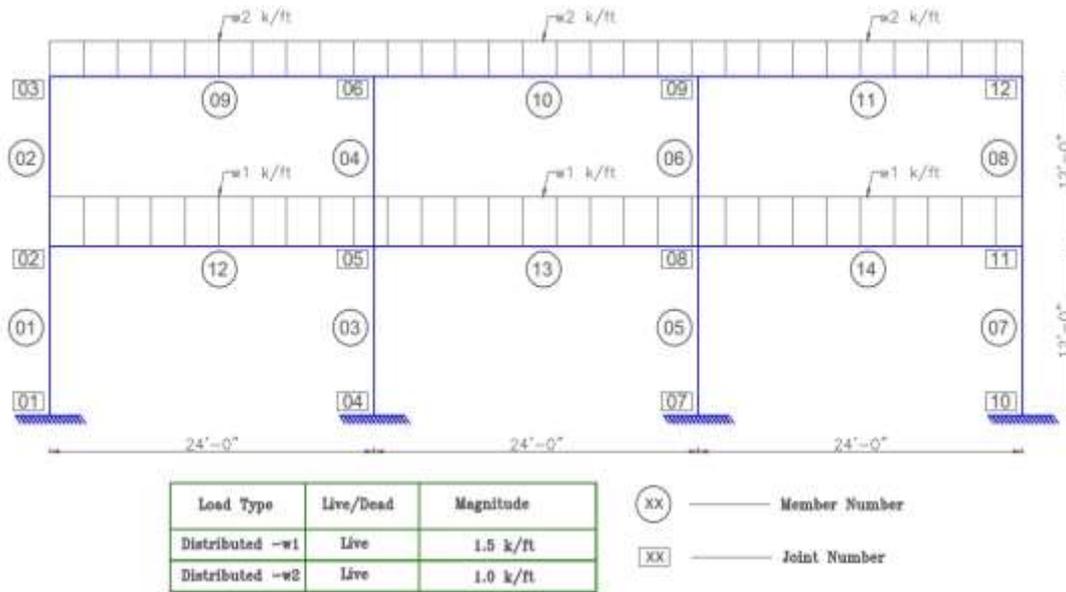


Figure 1 Geometry and load magnitudes for 3-bay 2-storey concrete frame structure

The algorithm attempts to create complete design profiles for all column and beam members. The design profiles are created for all beam and column members. These design profiles are created by evaluating all the feasible design values until the termination criterion is reached. The algorithm has the capability to optimize any plane concrete frame structure with any practical value of loading. The design variables plus the steel reinforcement corresponding to the minimum objective function value in the design profile are selected as local optimum for the corresponding member. The optimal design variables are assigned to each member as shown in Figure 2.

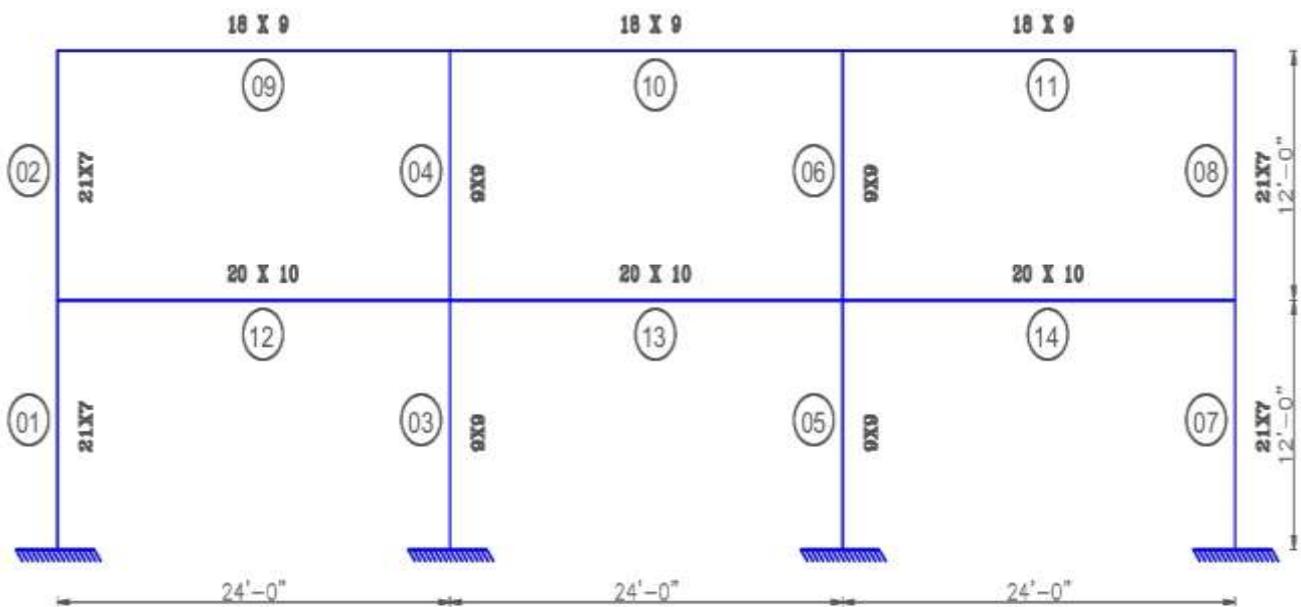


Figure 2 Final optimized member sections for beams and columns of 3-bay 2-storey concrete frame structure

The optimal values of member depth, width, amount of reinforcement, volume of concrete and the objective function for all the member are shown in Figure 3. The total steel used in a member is the algebraic sum of all the negative and positive steel areas converted to tons units. Whereas the total concrete used in a member is obtained by multiplying the area of cross section of the member to the length of the member. Finally the objective function is obtained by summing the products of total steel and concrete with their respective unit costs.

Member No.	Depth (in.)	Width (in.)	Total Steel (Ton)	Total Concrete (Cft)	Objective Function(Rs)
1	21	7	0.044	12.25	8522.84
2	21	7	0.044	12.25	8522.84
3	9	9	0.017	6.75	3815.40
4	9	9	0.017	6.75	3815.40
5	9	9	0.017	6.75	3815.40
6	9	9	0.017	6.75	3815.40
7	21	7	0.044	12.25	8522.84
8	21	7	0.044	12.25	8522.84
9	18	9	0.084	27	17148.93
10	18	9	0.087	27	17439.82
11	18	9	0.084	27	17143.16
12	20	10	0.110	33.33	21868.07
13	20	10	0.116	33.33	22585.21
14	20	10	0.110	33.33	21848.75

Figure 3 Detail of total steel reinforcement, concrete volume and cost for 3-bay 2-storey concrete frame structure

The design profiles of beam number 12 and Column number 2 are shown in Figure 4 and Figure 5 respectively. In both design profiles the final converged optimum design is obtained after 4 successive iterations. The blue line in both these figures shows the initial design profile of the members. Initial design profile of the member is the design profile when the surrounding elements are not yet assigned the optimal values. The initial optimal set of design variables is obtained from this initial profile and is assigned to the member. After which the algorithm move to the next member in the frame structure. In the remaining successive iterations, the algorithm search for the changes reflected in the optimal values of the design variables of each member, due to changing connected members. These changes are reflected in the form of changed cross-sectional dimensions of the member or the design area of steel or both, which changes the objective function. In this example, the changes in the members ceased until fourth iteration which is called the final converged profile of the member and is shown by the yellow line in these figures.

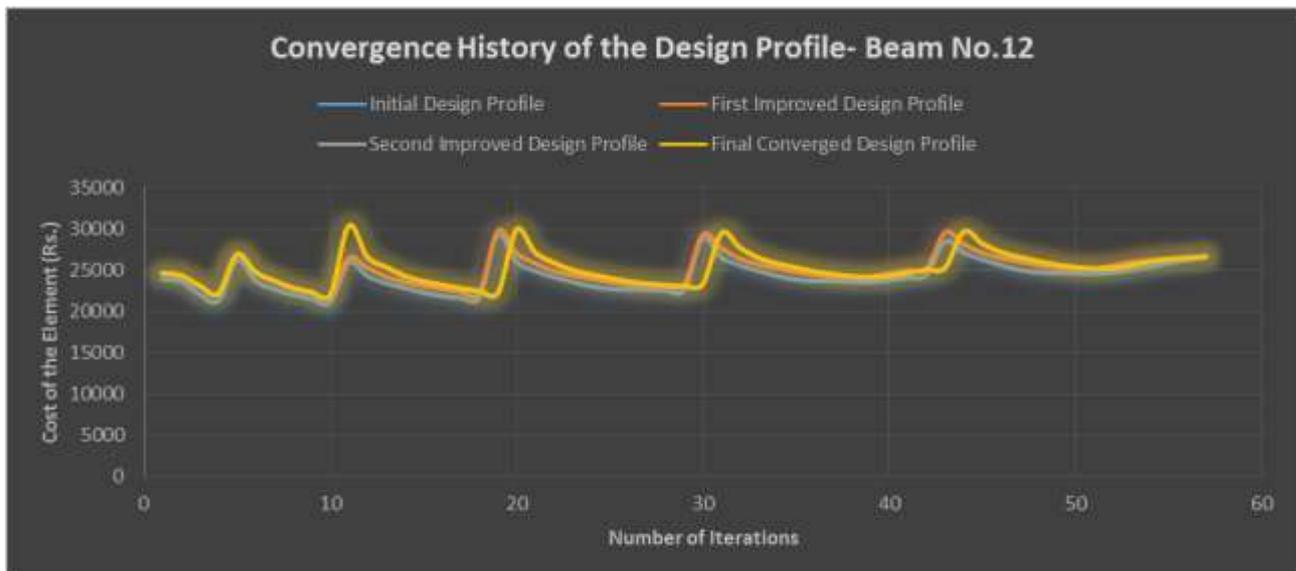


Figure 4 Convergence history of the design profile for beam number 12 of 3-bay 2-storey concrete frame structure

The final optimal design is obtained from the final converged design profile of the member. Both these figures indicate that the algorithm terminates the design profile when no lesser objective function value is possible. The sequential increase of the objective function is due to the arrangement of the value of 'r' in eq. (1) and eq. (3). This sequential arrangement makes it is possible to evaluate all the feasible designs for the search of optimal design variables for each member of the structure.

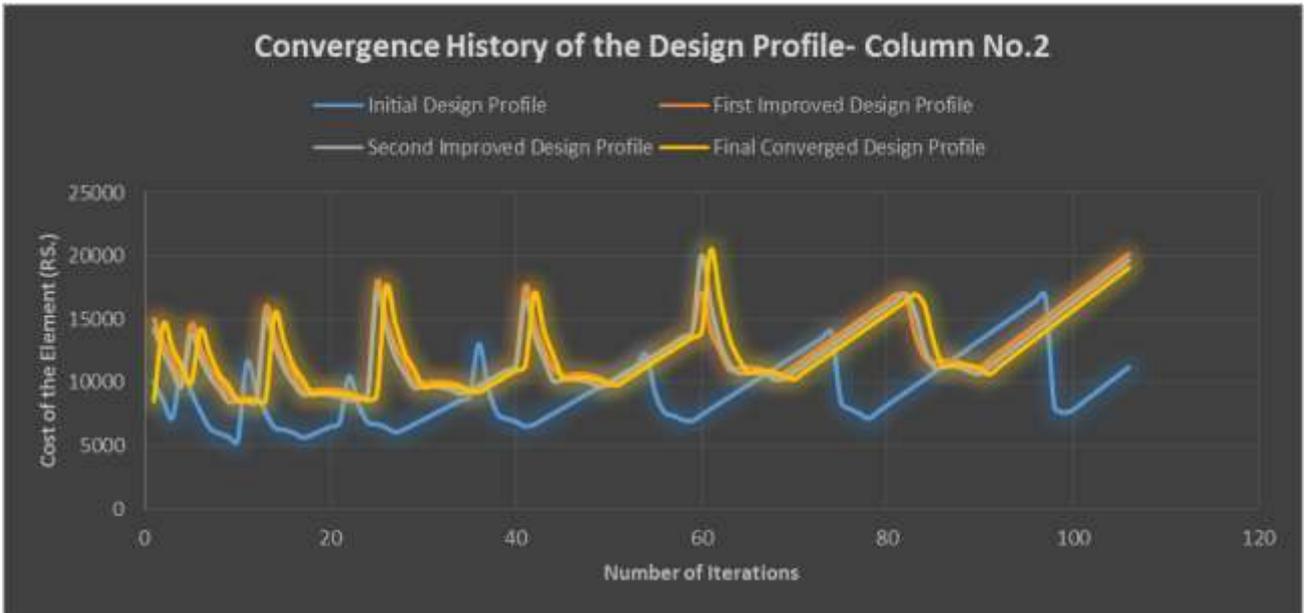
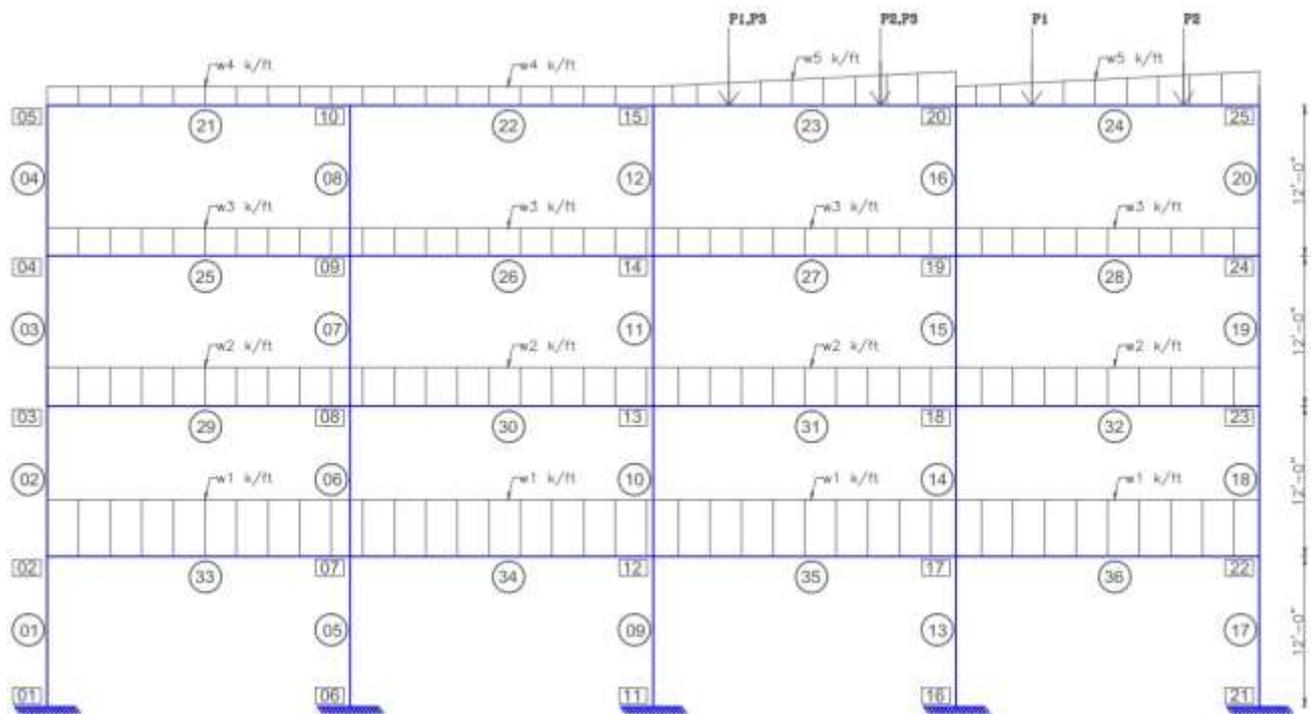


Figure 5 Convergence history of the design profile for Column number 2 of 3-bay 2-storey concrete frame structure

**Example 2: Four-Bay Four-Storey Plane Frame**

A four-bay, four-storey concrete rigid joint frame shown in Figure 6 is also optimized to test the performance of the developed algorithm for different types of loadings. For this problem the cost of steel is taken as 1, 10,000 Rs/Ton whereas the cost of concrete is taken as 230 Rs/Cft .The concrete compressive strength is taken as 3000psi whereas the yield strength of steel is taken as 40,000 psi. In this problem the members are subjected to both dead and live loadings. Three types of loadings i.e. pointed, uniformly distributed and uniformly varying loading are introduced with relatively less magnitudes. The length of each beam member is 24ft whereas the height of each column member is 12ft. The skeletal geometry of the structure and the load magnitudes for this problem are shown in Figure 6. The Figure 7 also shows the member and node numbers.



Load Type	Live/Dead	Magnitude
Pointed -P1	Dead	3.0 Kips
Pointed -P2	Dead	2.0 kips
Pointed -P3	Live	2.0 Kips
Distributed -w1	Live	0.15 k/ft
Distributed -w2	Live	0.08 k/ft
Distributed -w3	Live	0.04 k/ft
Distributed -w4	Live	0.01 k/ft
Distributed -w5	Live	0.01 kip at Start 0.02 kip at End

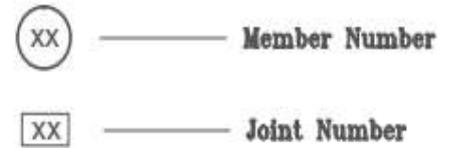


Figure 6 Geometry and load magnitudes for 4-bay 4-storey concrete frame structure

The beam members of storeys 1-3 and the beam number 21 and 22 of top most storey are subjected to only live uniformly distributed loading with varying magnitude for each storey. Whereas the beam number 23 and 24 of top most storey are subjected to live uniformly distributed, live uniformly varying as well as live and dead point loads. The same load combinations as example number 1 are used which are given below:

$$1.4D \quad \text{(ACI 9-1)}$$

$$1.2D + 1.6L \quad \text{(ACI 9-2)}$$

Complete design profiles for all column and beam members are created automatically by evaluating all the feasible design values until the termination criterion is reached. The design variables plus the steel reinforcement corresponding to the minimum objective function value in the design profile are selected as local optimum for the corresponding member. The optimal design variables are assigned to each member as shown in Figure 7.

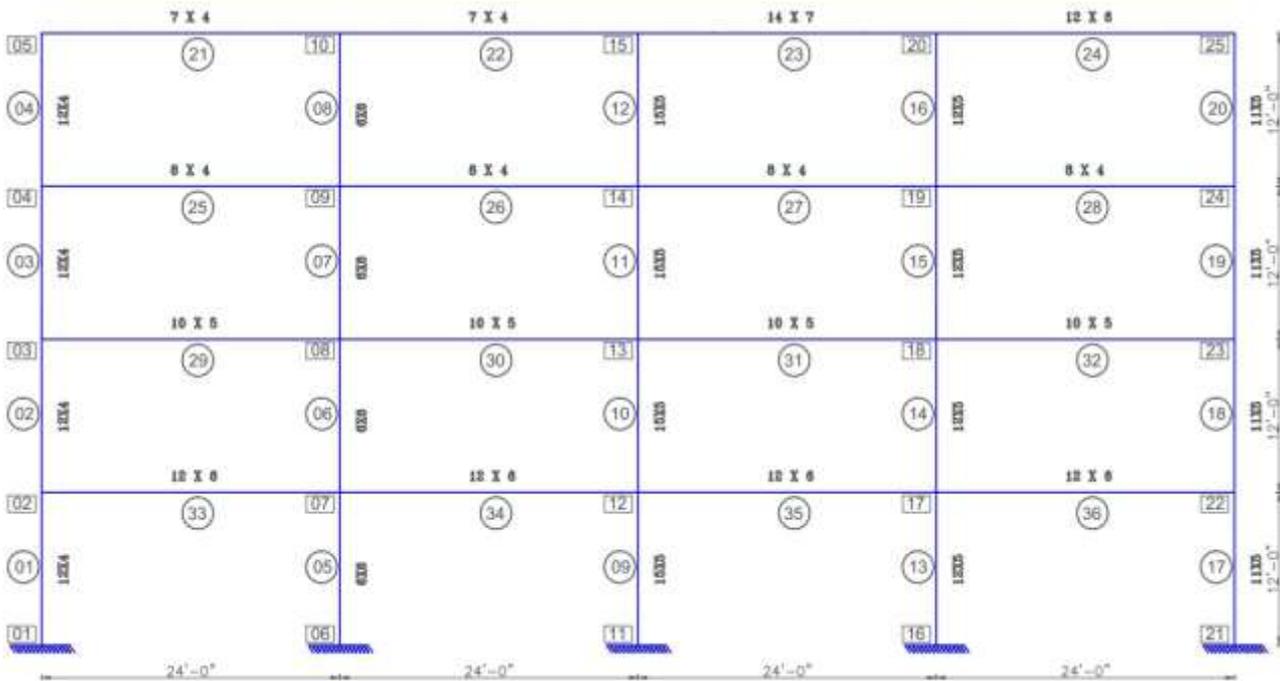


Figure 7 Final optimized member sections for beams and columns of 4-bay 4-storey concrete frame structure

The optimal values of member depth, width, amount of reinforcement, volume of concrete and the objective function for all the members are shown in Figure 8. The value of the objective function for each member is obtained by summing the product of total steel and total concrete with their respective unit costs.

Member No.	Depth (in.)	Width (in.)	Total Steel (Ton)	Total Concrete (Cft)	Objective Function(Rs)
1	12	4	0.011	4	2117.94
2	12	4	0.011	4	1141.79
3	12	4	0.011	4	1031.13
4	12	4	0.011	4	765.47
5	6	6	0.008	3	1516.90
6	6	6	0.008	3	1053.40
7	6	6	0.008	3	674.18
8	6	6	0.008	3	674.18
9	15	5	0.016	6.25	2022.54
10	15	5	0.016	6.25	1516.90
11	15	5	0.016	6.25	1474.78
12	15	5	0.016	6.25	3160.22
13	12	5	0.012	5	2064.67
14	12	5	0.012	5	2022.54
15	12	5	0.012	5	1769.72
16	12	5	0.012	5	2528.17
17	11	5	0.011	4.58	2317.49
18	11	5	0.011	4.58	1053.40
19	11	5	0.011	4.58	1550.27
20	11	5	0.011	4.58	1160.24
21	7	4	0.011	4.67	2900.37
22	7	4	0.012	4.67	2455.26
23	14	7	0.062	16.33	10642.50
24	12	6	0.047	12	7976.19
25	8	4	0.019	5.33	3498.07
26	8	4	0.020	5.33	3460.52
27	8	4	0.020	5.33	3514.83
28	8	4	0.018	5.33	3299.00
29	10	5	0.027	6.33	4920.96
30	10	5	0.027	6.33	4993.14
31	10	5	0.026	6.33	4851.52
32	10	5	0.027	6.33	4868.53
33	12	6	0.036	12	6748.42
34	12	6	0.037	12	6915.41
35	12	6	0.037	12	6868.62
36	12	6	0.036	12	6774.55

Figure 8 Detail of total steel reinforcement, concrete volume and cost for 4-bay 4-storey concrete frame structure

The design profiles of beam number 34 and Column number 05 are shown in Figure 9 and Figure 10 respectively. In both design profiles the final converged optimum design is obtained after 4 successive iterations. The blue line in both these figures show the initial design profile of the member. Initial design profile of the member is the design when the surrounding elements are not yet assigned the optimal values.

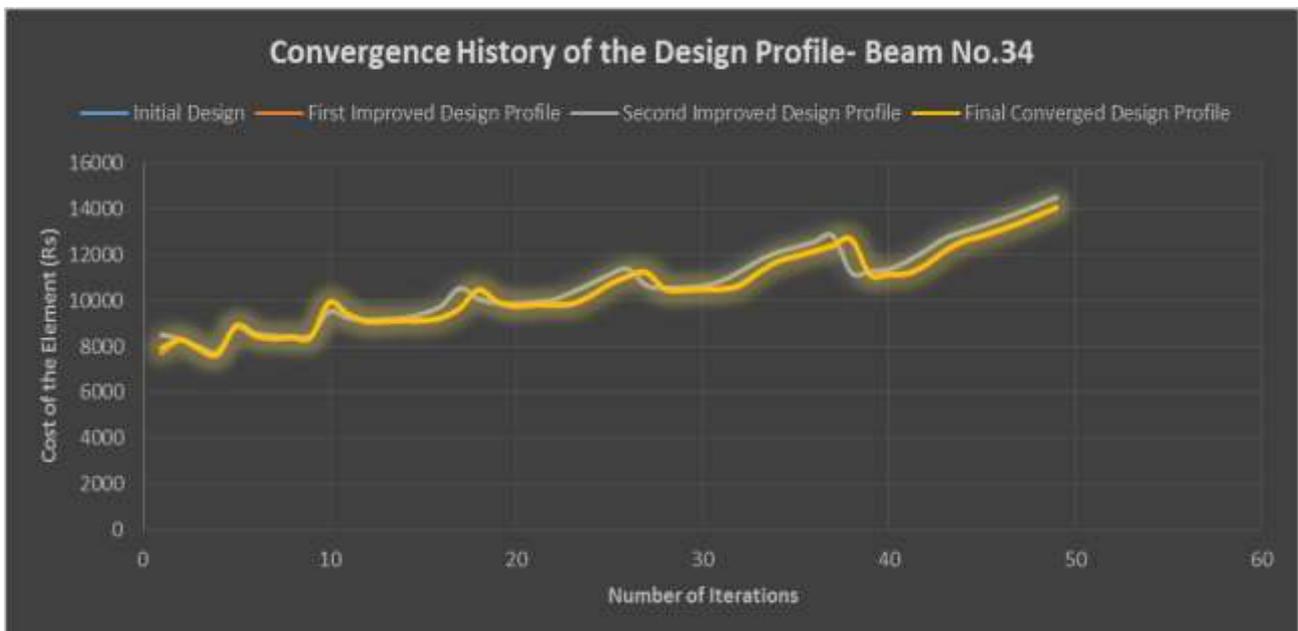
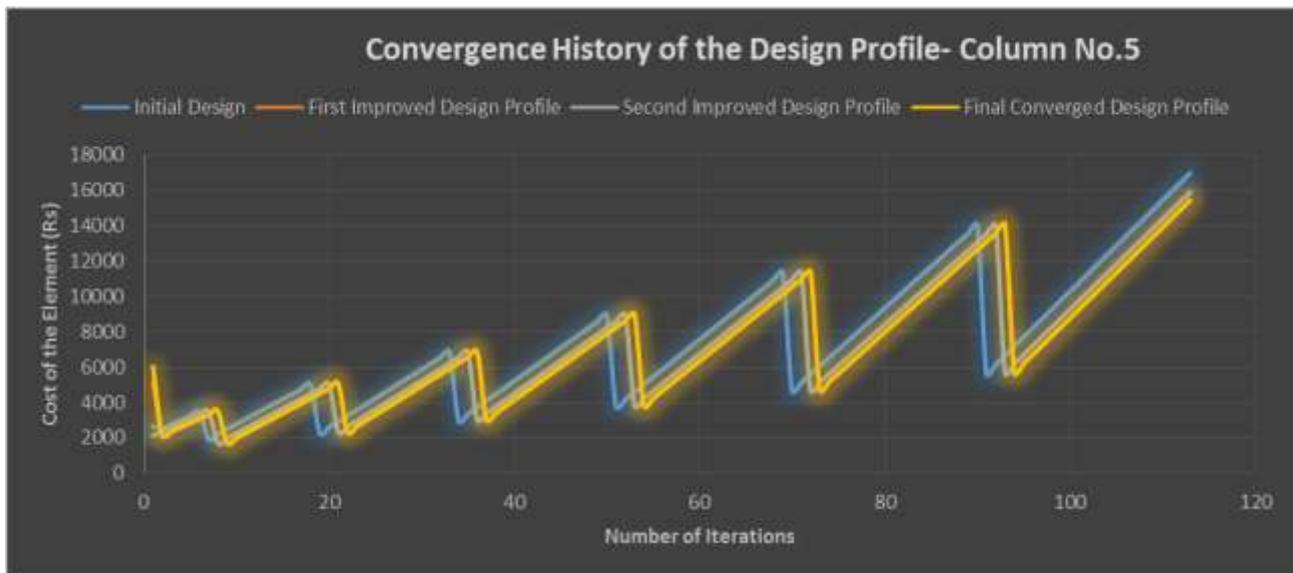


Figure 9 Convergence history of the design profile for beam number 34 of 4-bay 4-storey concrete frame structure

The final optimal design is obtained after considering the effects of the surrounding changing connected elements for each member until the values converged to constant design parameters and is indicated by the yellow line in these figures. Both these figures indicate that the algorithm terminates the design profile when no lesser objective function value is possible.



**Figure 10** Convergence history of the design profile for Column number 5 of 4-bay 4-storey concrete frame structure

## VI. Conclusion:

This study aims at introducing an efficient fully automated optimization approach based on ultimate strength design for moment-resisting concrete frame problems. The method creates design profiles covering all the feasible designs for each frame member. The optimization of a plane concrete frame structure is a complex problem as the design variables are closely inter dependent. The design of each member is also affected by the surrounding connected members.

This method solve the problem by integrating two modules i.e. the module for checking the design constraints and the module for creating design profile. In the first phase an initial design profile is created for all the members and the initial optimum design is assigned to all members. In the second phase consequent improved design profiles are created considering the effects of surrounding members for each member until a final converged design profile is obtained. The design variables corresponding to the minimum objective function in the final converged design profile are thus selected as local optimum for each member. An important feature of this method is that it search the optimal design in a design space which contain all the practical feasible designs.

This method accommodates multiple loading conditions and can optimized plane concrete frames of any number of members and with any type of live and dead load combination. Moreover, frames with inclined members can also be optimized using this algorithm. This procedure enables the designer to evaluate all the practical feasible designs with minimum effort and thus to choose the design which best suits the local conditions.

Three dimensional problem can also be introduced by extending the analysis and design feature of the algorithm only. Similarly, other design codes can be introduced by making slight modifications in the modules.

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