DECISION MAKING PROBLEM IN PENTAGONAL INTUITIONISTIC FUZZY NUMBER USING TOPSIS METHOD

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ABSTRACT:- Decision making problem is the process of finding the best option from all the feasible alternatives. In this paper a solution methodology for Multi Criteria Decision making problems in pentagonal intuitionistic fuzzy numbers. Therefore the objective of this paper is to select a best student on the basis of the student’s performance using ranking techniques for pentagonal intuitionistic fuzzy numbers can be converted into crisp value with the help of this value using TOPSIS method we get the best student of college.

KEYWORDS: Fuzzy numbers, intuitionistic fuzzy numbers, pentagonal intuitionistic fuzzy number, accuracy function, topsis.

1. INTRODUCTION

In 1986 Intuitionistic fuzzy set was introduced by Atanassov. In this paper we introduced pentagonal Intuitionistic Fuzzy Number which is included membership & non-membership value. Under some condition, the accuracy function is defuzzified into crisp value by adding both membership & non-membership values, which is less than one. Technique for order performance by similarity to ideal solution (Topsis), one of knows classical MCDM method was first developed by Hwang & yoon for solving MCDM problem. The basic concept of topsis method is to select the best alternatives. C.T. Chen extended the concept of topsis to develop a methodology for solving MCDM. In this paper, the objective is to select a best student on the basis of the student’s performance using ranking techniques for pentagonal intuitionistic fuzzy numbers then the value can be converted into crisp value, with the help of this value using TOPSIS method we get the best student of college.

2. PRELIMINARIES

2.1 Definition (Fuzzy Set) [7]
Let A be a classical set, μₐ(x) be a function from A to [0, 1]. A fuzzy set A* with the membership function μₐ(x) is defined by A* = {(x, μₐ(x)); x ∈ A and μₐ(x) ∈ [0, 1]}.

2.2 Definition (Intuitionistic Fuzzy Set)[2]
Let X be a non empty set. An intuitionistic fuzzy set A in X is an object having the form A is equal to {(x, μₐ(x)), νₐ(x)); x ∈ X}, where the function μₐ(x), νₐ(x): X→[0, 1] define respectively, the degree of membership and the degree of non-membership of the element x ∈ X to the set A, which is a subset of X, and for every element X ∈ X, 0 ≤ μₐ(x) + νₐ(x) ≤ 1.

2.3 Definition (Fuzzy Number) [7]
A fuzzy number A is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by ℱ.

2.4 Definition (Pentagonal Fuzzy Number) [5]
A pentagonal fuzzy number (PFN) of a fuzzy set A is defined as A_p = {a, b, c, d, e}, and its membership function is given by
\[ \mu_{A_1}(x) = \begin{cases} 
0 & \text{for } x < a_1, \\
(x-a_1) & \text{for } a_1 \leq x \leq b_1 \\
(x-b_1) & \text{for } b_1 \leq x \leq c_1 \\
1 & \text{for } x = c_1 \\
(d_1-c_1) & \text{for } c_1 \leq x \leq d_1 \\
(e_1-d_1) & \text{for } d_1 \leq x \leq e_1 \\
0 & \text{for } x > e_1 
\end{cases} \]

\[ \vartheta_{A_1}(x) = \begin{cases} 
1 & \text{for } x < a_1, \\
(b_2-a_2) & \text{for } a_2 \leq x \leq b_2 \\
(c_2-b_2) & \text{for } b_2 \leq x \leq c_2 \\
0 & \text{for } x = c_1 \\
(d_2-c_2) & \text{for } c_2 \leq x \leq d_2 \\
(e_2-d_2) & \text{for } d_2 \leq x \leq e_2 
\end{cases} \]

### 2.5 Definition (Pentagonal Intuitionistic Fuzzy Number)[5,2]

A pentagonal intuitionistic fuzzy number \( A_1 \) of an intuitionistic fuzzy set is defined as \( A_1 = \{(a_1, b_1, c_1, d_1, e_1), (a_2, b_2, c_2, d_2, e_2)\} \) where all \( a_1, b_1, c_1, d_1, e_1, a_2, b_2, c_2, d_2, e_2 \) are real numbers and its membership function \( \mu_{A_1}(x) \), non-membership function \( \vartheta_{A_1}(x) \), are given by

3. ACCURACY FUNCTION OF AN INTUITIONISTIC PENTAGONAL FUZZY NUMBER

Accuracy function of a pentagonal intuitionistic fuzzy number \( A_1 = \{(a_1, b_1, c_1, d_1, e_1), (a_2, b_2, c_2, d_2, e_2)\} \) is defined as \( H(A_1) = (a_1 + a_2 + b_1 + b_2 + c_1 + c_2 + d_1 + d_2 + e_1 + e_2)/5 \).

4. PROCEDURE

**STEP 1**
Using accuracy function of Pentagonal Intuitionistic fuzzy number, fuzzy value can be converted into crisp value.
STEP 2
Construct the decision matrix. The first step of the TOPSIS method involves the construction of a decision matrix (DM)

\[
\text{DM} = \begin{bmatrix}
X_{11} & \ldots & X_{1n} \\
\vdots & \ddots & \vdots \\
X_{m1} & \ldots & X_{mn}
\end{bmatrix}
\]

Where ‘i’ is the criterion index \(i = (1, \ldots, m^2)\), \(m\) is the number of potential sites. ‘j’ is the alternative index \((j=1, \ldots, n)\) the elements \(c_1, c_2, \ldots, c_n\) refer to criteria. While \(L_1, L_2, \ldots, L_n\) refer to the alternative locations.

STEP 3
Construct to the normalized decision matrix.

\[
r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}^2}} ; \text{ i=1, 2, \ldots, n and j=1,2\ldotsm}
\]

Where \(x_{ij}\) and \(r_{ij}\) are original and the normalized score of decision matrix respectively.

STEP 4
Construct the weighted normalized decision matrix by normalized decision matrix by multiplying the weights \(w_i\) of evaluation criteria with the normalized decision matrix \(r_{ij}\).

\[
V_{ij} = w_j x r_{ij} \quad j= 1,2,3\ldots m \\
i = 1,2,3\ldots n
\]

STEP 5
Determine the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS)

\[
A^+ = \{V_1^+, V_2^+, \ldots, V_n^+\} \text{ maximum values}
\]

Where \(V_i^+ = \{\max(V_{ij}) \text{ if } j \in J; \min(V_{ij}) \text{ if } j \in J^-\}\)

\[
A^- = \{V_1^-, V_2^-, \ldots, V_n^-\} \text{ minimum values}
\]

Where \(V_i^- = \{\min(V_{ij}) \text{ if } j \in J; \max(V_{ij}) \text{ if } j \in J^-\}\)

STEP 6
Calculate the separation measures of each alternative from PIS and NIS

\[
s_i^+ = \sqrt{\sum_{j=1}^{n} (V_j^+ - V_{ij})^2} \quad i=1,2,3\ldots m
\]

\[
s_i^- = \sqrt{\sum_{j=1}^{n} (V_j^- - V_{ij})^2} \quad i=1,2,3\ldots m
\]

STEP 7
Calculate the relative closeness co efficient to the ideal solution of each alternative

\[
C_i = \frac{s_i^-}{s_i^+ + s_i^-} , \quad 0 \leq C_i \leq 1
\]

\[
i = 1, 2 \ldots m
\]

STEP 8
Based on the decreasing values of closeness coefficient, alternatives are ranked from most valuable to worst. The alternative having highest closeness coefficient \((c_i)\) is selected.

5. NUMERICAL EXAMPLE

A teacher is desirable to select the best student on the basis of choice parameter. After pre-evaluation four students \(A_i\) (i=1,2,3,4) have remind as alternatives for further evaluation. Four criteria are considered as \(B_1;\) attendance \(B_2;\)
communication skill $B_3$; percentage $B_4$; sports skill. (Whose weighting vector is completely unknown) they construct the following table

<table>
<thead>
<tr>
<th>PERFORMANCE</th>
<th>ATTENDANCE</th>
<th>COMMUNICATION SKILLS</th>
<th>PERCENTAGE</th>
<th>SPORTS SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>STUDENTS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABDUL</td>
<td>${(3,4,5,6,7); (4,5,6,7,8)}$</td>
<td>${(4,6,7,8,9); (7,9,12,13,15)}$</td>
<td>${(2,10,12,13,14); (7,10,11,12,14)}$</td>
<td>${(2,4,6,8,9); (7,8,9,10,12)}$</td>
</tr>
<tr>
<td>ILYAAS</td>
<td>${(2,3,5,6,7); (3,4,5,6,9)}$</td>
<td>${(8,9,10,12); (9,11,13,15,17)}$</td>
<td>${(6,8,14,15,16); (8,9,13,15,16)}$</td>
<td>${(4,5,6,7,8); (5,6,7,8,9)}$</td>
</tr>
<tr>
<td>SHEIK ISMAIL</td>
<td>${(5,7,10,11,12); (6,7,8,9,10)}$</td>
<td>${(1,4,5,6,8); (8,9,10,11,13)}$</td>
<td>${(7,9,12,13,14); (4,6,10,12,13)}$</td>
<td>${(2,4,7,8,9); (7,8,9,10,11)}$</td>
</tr>
<tr>
<td>SULTHAN</td>
<td>${(5,6,7,8,9,11); (10,13,15,17)}$</td>
<td>${(2,3,4,5,6); (5,6,7,8,9)}$</td>
<td>${(7,11,13,14,16); (10,12,13,14,15)}$</td>
<td>${(4,8,10,11,12); (7,8,9,10,11)}$</td>
</tr>
</tbody>
</table>

**STEP 1**
Using accuracy function of Pentagonal Intuitionistic fuzzy number, fuzzy value can be converted into crisp value.

**STEP 2**
Calculate $(\Sigma x_{ij}^2)^{1/2}$ for each column and divide each column by that to get $r_{ij}$
Then it is multiplied weight criteria. Therefore it is

\[
V_{11} = 0.1 \times 0.38 = 0.038 \\
V_{12} = 0.2 \times 0.51 = 0.102 \\
V_{13} = 0.3 \times 0.46 = 0.138 \\
V_{14} = 0.4 \times 0.48 = 0.192
\]

**STEP 4**
Find the positive ideal solution \( A^* \).
\( A^* = \{0.062, 0.132, 0.165, 0.232\} \)

<table>
<thead>
<tr>
<th>PERFORMANCE</th>
<th>ATTENDENCE</th>
<th>COMMUNICATION SKILLS</th>
<th>PERCENTAGE</th>
<th>SPORTS SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABDUL</td>
<td>0.38</td>
<td>0.51</td>
<td>0.46</td>
<td>0.48</td>
</tr>
<tr>
<td>ILYAAS</td>
<td>0.31</td>
<td>0.66</td>
<td>0.53</td>
<td>0.42</td>
</tr>
<tr>
<td>SHEIK ISMAIL</td>
<td>0.58</td>
<td>0.43</td>
<td>0.44</td>
<td>0.48</td>
</tr>
<tr>
<td>SULTHAN</td>
<td>0.62</td>
<td>0.31</td>
<td>0.55</td>
<td>0.58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STUDENTS</th>
<th>PERFORMANCE</th>
<th>ATTENDENCE</th>
<th>COMMUNICATION SKILLS</th>
<th>PERCENTAGE</th>
<th>SPORTS SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABDUL</td>
<td>0.0005</td>
<td>0.0009</td>
<td>0.0007</td>
<td>0.0016</td>
<td></td>
</tr>
<tr>
<td>ILYAAS</td>
<td>0.0009</td>
<td>0</td>
<td>0.00003</td>
<td>0.0040</td>
<td></td>
</tr>
<tr>
<td>SHEIK ISMAIL</td>
<td>0.00001</td>
<td>0.0021</td>
<td>0.0010</td>
<td>0.0016</td>
<td></td>
</tr>
<tr>
<td>SULTHAN</td>
<td>0</td>
<td>0.0049</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

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STEP 5
Find the negative ideal solution $A^-$. 

$A^- = \{0.031, 0.062, 0.132, 0.168\}$

<table>
<thead>
<tr>
<th>STUDENTS</th>
<th>PERFORMANCE</th>
<th>ATTENDANCE</th>
<th>COMMUNICATION SKILLS</th>
<th>PERCENTAGE</th>
<th>SPORTS SKILLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABDUL</td>
<td>0.007</td>
<td>0.0016</td>
<td>0.00003</td>
<td>0.0005</td>
<td></td>
</tr>
<tr>
<td>ILYAAS</td>
<td>0</td>
<td>0.0049</td>
<td>0.0007</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>SHEIK ISMAIL</td>
<td>0.0007</td>
<td>0.0005</td>
<td>0.0007</td>
<td>0.0005</td>
<td></td>
</tr>
<tr>
<td>SULTHAN</td>
<td>0.0009</td>
<td>0</td>
<td>0.0010</td>
<td>0.004</td>
<td></td>
</tr>
</tbody>
</table>

STEP 6(a)
Determine the separation from ideal solution $S_i^+$

<table>
<thead>
<tr>
<th>STUDENTS</th>
<th>$\Sigma(v_j^+ - v_{ij})^2$</th>
<th>$S_i^+ = [\Sigma(v_j^+ - v_{ij})^2]^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABDUL</td>
<td>0.0038</td>
<td>0.061</td>
</tr>
<tr>
<td>ILYAAS</td>
<td>0.0050</td>
<td>0.071</td>
</tr>
<tr>
<td>SHEIK ISMAIL</td>
<td>0.0048</td>
<td>0.069</td>
</tr>
<tr>
<td>SULTHAN</td>
<td>0.0049</td>
<td>0.07</td>
</tr>
</tbody>
</table>

STEP 6 (b)
Determine the separation from ideal solution $S_i^-$

<table>
<thead>
<tr>
<th>STUDENTS</th>
<th>$\Sigma(v_j^- - v_{ij})^2$</th>
<th>$S_i^- = [\Sigma(v_j^- - v_{ij})^2]^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABDUL</td>
<td>0.0092</td>
<td>0.095</td>
</tr>
<tr>
<td>ILYAAS</td>
<td>0.0056</td>
<td>0.075</td>
</tr>
<tr>
<td>SHEIK ISMAIL</td>
<td>0.0026</td>
<td>0.051</td>
</tr>
<tr>
<td>SULTHAN</td>
<td>0.0061</td>
<td>0.078</td>
</tr>
</tbody>
</table>
STEP 7
Calculate the relative closeness to the ideal solution \( C_i^+ = \frac{S_i}{(S_i^+ + S_i^-)} \)

<table>
<thead>
<tr>
<th></th>
<th>( S_i/(S_i^+ + S_i^-) )</th>
<th>( C_i^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABDUL</td>
<td>0.095/0.156</td>
<td>0.6089</td>
</tr>
<tr>
<td>ILYAAS</td>
<td>0.075/0.146</td>
<td>0.513</td>
</tr>
<tr>
<td>SHEIK ISMAIL</td>
<td>0.051/0.12</td>
<td>0.425</td>
</tr>
<tr>
<td>SULTHAN</td>
<td>0.078/0.148</td>
<td>0.527</td>
</tr>
</tbody>
</table>

Therefore, we concluded that abdul is the best student for this TOPSIS method.

6. CONCLUSION

In this paper focus on MCDM Intuitionistic Fuzzy environment problems. In which the rating of alternatives are represented as pentagonal intuitionistic fuzzy numbers. The accuracy of ranking method is developed for the MCDM and applied to TOPSIS method with pentagonal intuitionistic fuzzy numbers. With the help of derived result we conclude the teacher can select the best student by using their choice factors.

7. REFERENCES