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DYNAMICS IN A FRACTIONAL ORDER PARALLEL RLC CIRCUIT

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Abstract — *This paper considers a two dimensional fractional order model of parallel RLC circuit. Existence of equilibrium points is established and local stability conditions are obtained. The phase portraits are obtained for different sets of parameter values. Numerical simulations are performed.*

Keywords- *Fractional order,Mathematical Model of RLC circuit, Numerical Simulation, Stability*

I. INTRODUCTION

The Components of an electric circuit is connected in many ways, simplest of these are series and parallel. In parallel circuits, connections between the components are made in multiple path. They are connected in such a way that voltage applied is same to each component. The general form of differential equation describing a series RLC circuit and Parallel RLC are identical due to the fact that the parallel RLC is the dual impedence of a series RLC. Figure-(1) is a parallel RLC circuit.

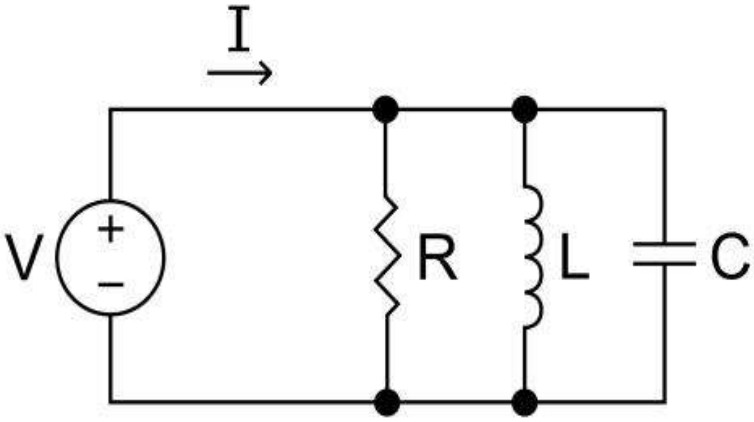

Figure 1: Parallel RLC circuit

Section 2 of the paper describes the Fractional order model of RLC circuit with parallel connections of the components with examples for different motions. Section 3 explains the discrete fractional order model and followed with numerical examples. Section 4 is the conclusion of the paper.

II. SYSTEM OF FRACTIONAL DIFFERENTIAL EQUATIONS

Analysis of the model is done based on the fixed point of the system of equations and eigenvalues. Here we consider a RLC circuit with the components connected parallel to each other. In this section, we will discuss the effects of the fractional order on the oscillatory behavior of the system in the absence of the forcing term.

The Fractional order system representing a Parallel RLC circuit is given by

$$\begin{bmatrix} D^{\nu}\Phi(t) \\ D^{\nu}x(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\beta \end{bmatrix} \begin{bmatrix} \Phi(t) \\ x(t) \end{bmatrix}, \quad (1)$$

where $D^{\nu} = \frac{d^{\nu}}{dt^{\nu}}$, ν is the Fractional order and $\Phi(t)$ is the magnetic flux. The equilibrium point of the system (1) is $(0,0)$. The eigen values obtained from the Jacobian matrix for the system (1) are $\lambda_{1,2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$.

It is clear from the eigenvalues that relation between β and ω_0 plays a crucial role in determining the motion of the system (1). Here we analyse the motion of the system of fractional order with some numerical examples and simulations. In numerical simulations, the magnetic flux $\Phi(t)$ is taken along the horizantal axis and $x(t)$ along the vertical axis.

Example 1 *Consider the system*

$$\begin{bmatrix} D^{v}\Phi(t) \\ D^{v}x(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.49 & 0 \end{bmatrix} \begin{bmatrix} \Phi(t) \\ x(t) \end{bmatrix}, \quad \begin{bmatrix} \Phi(0) \\ x(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{2}$$

with v taking values $0.5, 0.6, 0.7, 0.8, 0.9, 0.99$, then the equilibrium points of the system (2) is clearly stable as shown in Figure-(2).

Figure- (3) explains the motion of the system (2) with $v = 1$. The system (2) has no damped motion since $\beta = 0$ and is less than ω_0. The increase in v for the system from 0.5 to 1 results in spiral motion moving inwards approaching the equilibrium point. The eigenvalues of the system (2) are $\lambda_{1,2} = \pm i\ 0.7$.

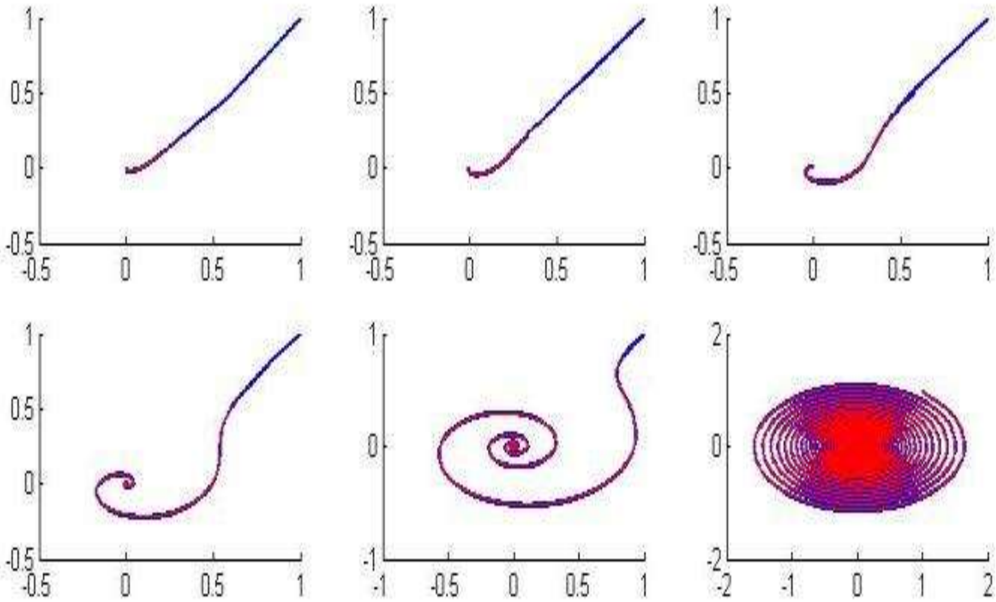

Figure 2: Phase Portrait of System (2)

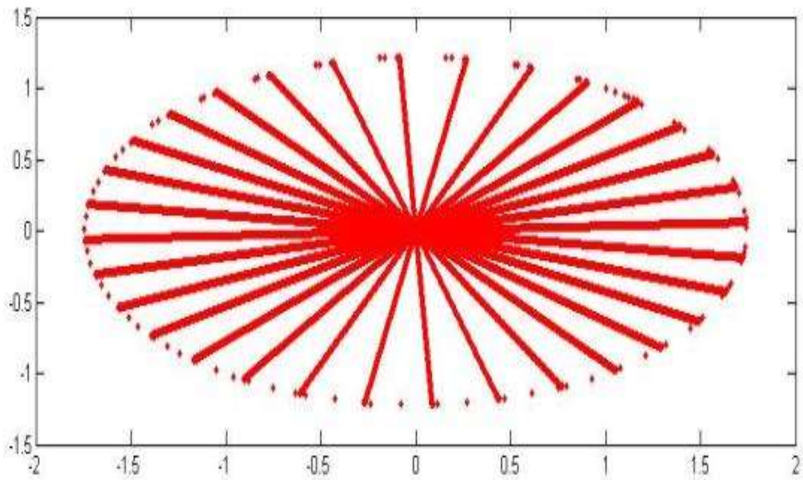

Figure 3: Phase Portrait System (2) of order 1

Example 2 *The system*

$$\begin{bmatrix} D^{v}\Phi(t) \\ D^{v}x(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.49 & -1.4 \end{bmatrix} \begin{bmatrix} \Phi(t) \\ x(t) \end{bmatrix}, \quad \begin{bmatrix} \Phi(0) \\ x(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{3}$$

with critically damped motion for v taking values $0.5, 0.6, 0.7, 0.8, 0.9, 0.99$ as in Figure-(7) is stable.

The system (3) with order 1 in Figure-(8) with $\beta = \omega_0$ has no oscillations when compared to motion of the system when the order is between 0.5 and 1. The eigenvalues of the system (3) are $\lambda_1 = -0.7 = \lambda_2$. The system becomes unstable for the order of the system less than or equal to 0.45.

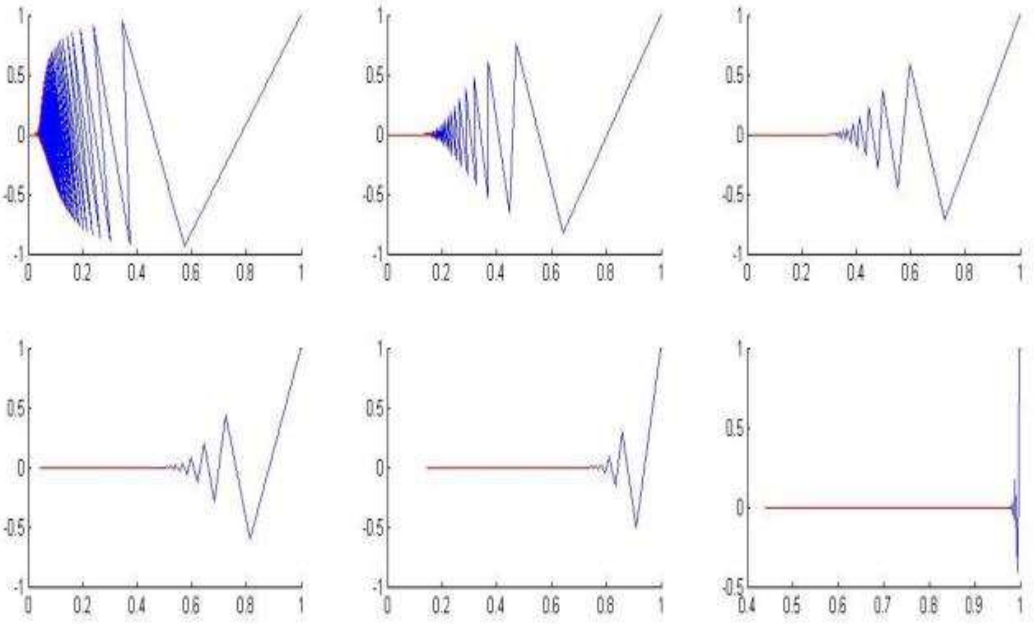

Figure 4: Phase Portrait of System (3)

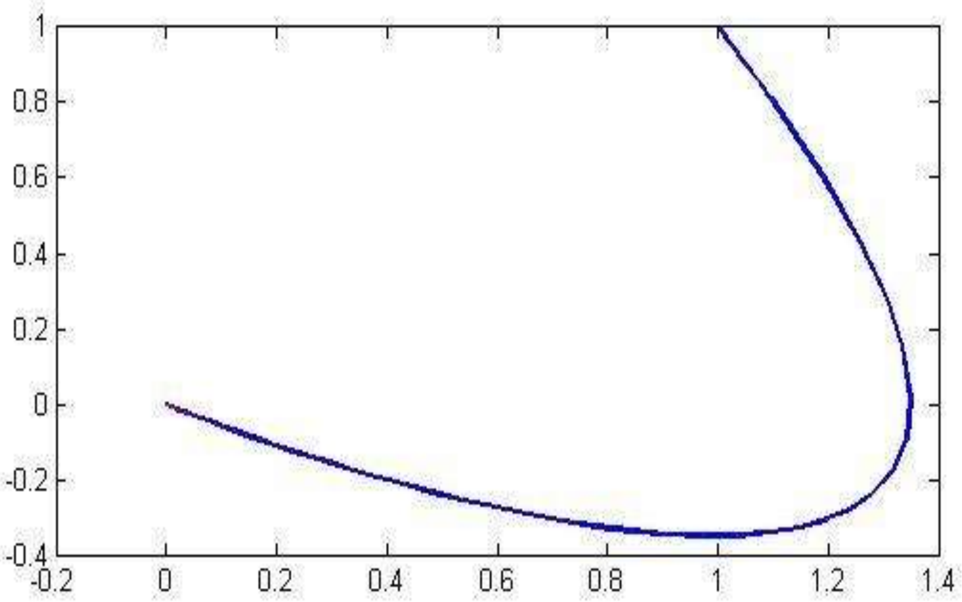

Figure 5: Phase Portrait of System (3) of order 1

The effects of the commensurate Fractional order has been explained in the above examples. Now, the following example explains the oscillatory behavior for the incommensurate order.

Example 3 *The order of the second equation in the system is fixed at 0.95 and the order of the first equation is varied between* 0.5 *and* 0.99

$$\begin{bmatrix} D^{v_1}\Phi(t) \\ D^{0.95}x(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -0.49 & -1.4\end{bmatrix}\begin{bmatrix}\Phi(t)\\ x(t)\end{bmatrix}, \quad \begin{bmatrix}\Phi(0)\\ x(0)\end{bmatrix}=\begin{bmatrix}1\\ 1\end{bmatrix} \tag{4}$$

then the system (4) has stable equilibrium points given by Figure -(9).

Thus, from Examples 2 and 3 for $\beta = \omega_0$ the effects of the commensurate and incommensurate fractional orders are clearly executed. The oscillations of the systems is reduced in case of incommensurate system (4) when compared to oscillations of the system (3). Also, in case of system(4) goes further close to the equilibrium point than the system (3).

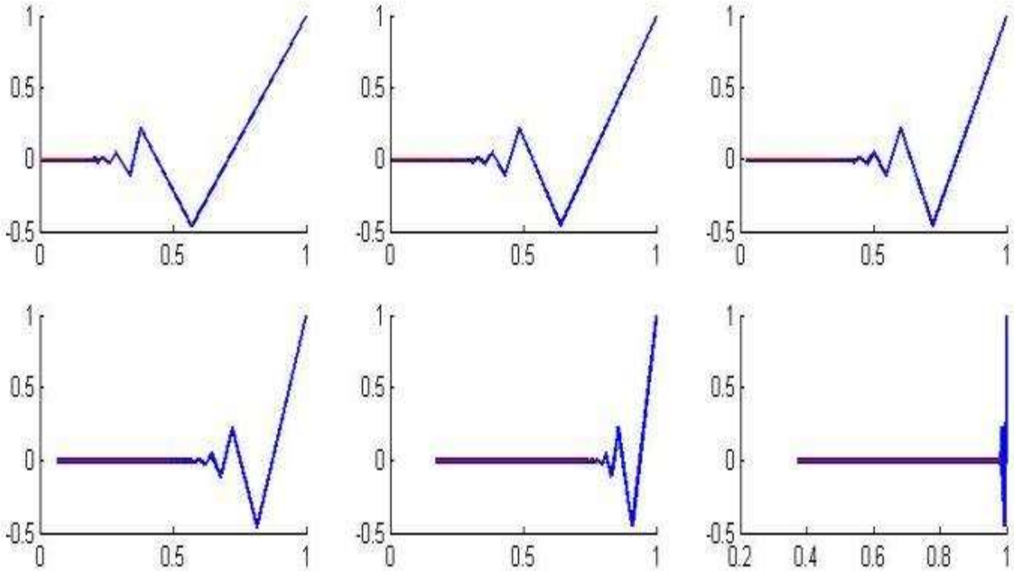

Figure 6: Phase Portraits of System (4)

III. DISCRETE FRACTIONAL ORDER SYSTEM

Discretisation of Equation (1) takes the form

$$\begin{bmatrix}\Phi(t+1)\\ x(t+1)\end{bmatrix} = \begin{bmatrix}1 & S\\ -S\ \omega_0^2 & 1-2S\ \beta\end{bmatrix}\begin{bmatrix}\Phi(t)\\ x(t)\end{bmatrix}, \quad (5)$$

where $S = \frac{h^\nu}{\Gamma(1+\nu)}$. The eigenvalues of the system (5) obtained from Jacobian matrix at the trivial equilibrium point $(0,0)$ are $\lambda_{1,2} = 1 - S\ \beta \pm \sqrt{\beta^2 - \omega_0^2}$, which in turn results in three different oscillations of the system.

Example 4 *Consider $\beta < \omega_0$, the Eigenvalues are complex, conjugate with system (5) having under damped oscillations. Let $R = 70, L = 7, C = 0.0238$ and $S = 0.001$ then the eigen values are $\lambda_{1,2} = -0.9997 \pm i\ 0.0024$. From the phase portrait, it is clear that the equilibrium point is stable as the spiral moves inward approaching the fixed point.Thus under damped oscillations of the system (5) reaches the equilibrium position.*

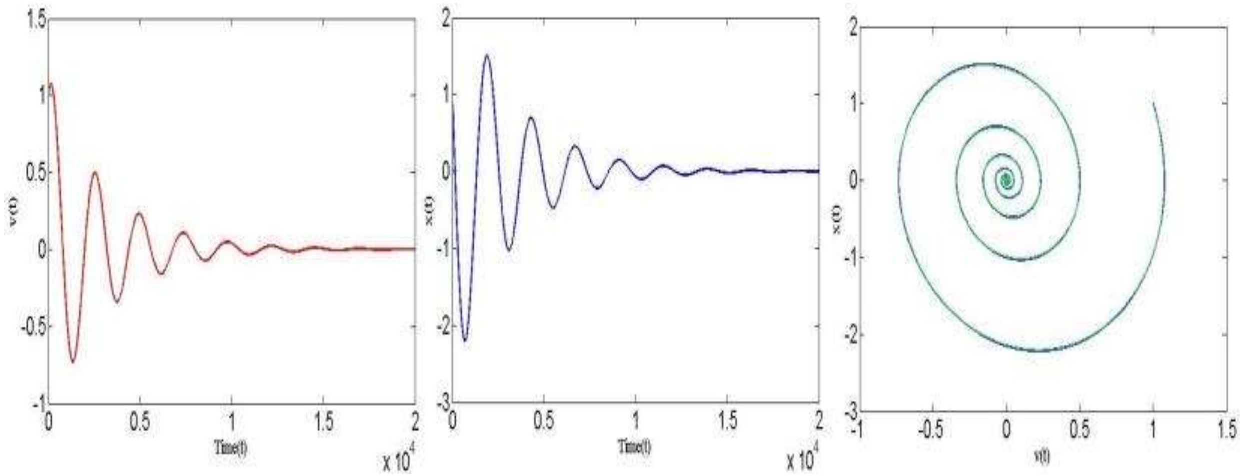

Figure 7: Under Damping

Example 5 *For the system having over damped oscillations, the eigenvalues are real and distinct. Take $R = 6, L = 7, C = 0.0238$ and $S = 0.001$ then the eigenvalues are $\lambda_1 = -0.9990$ and $\lambda_2 = -0.9940$. The Over damped oscillations of the system (5) shown in Figure-(8) approaches the equilibrium position as the time increases.*

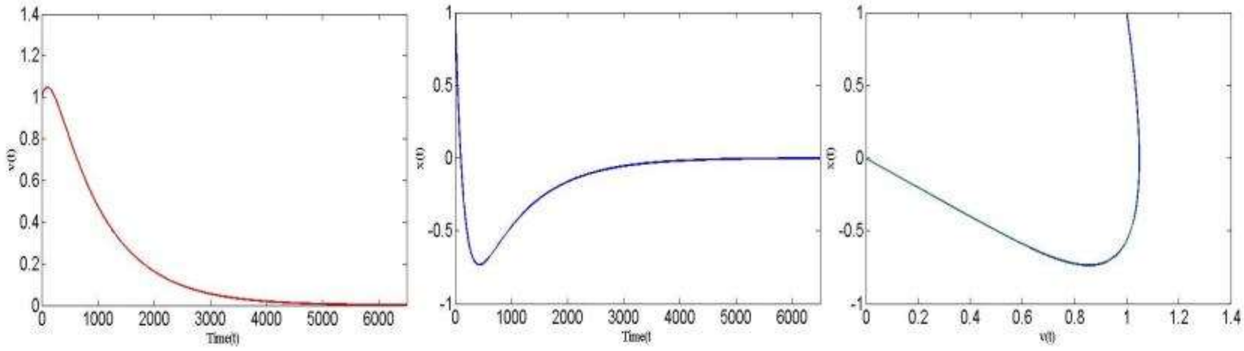

Figure 8: Over Damping

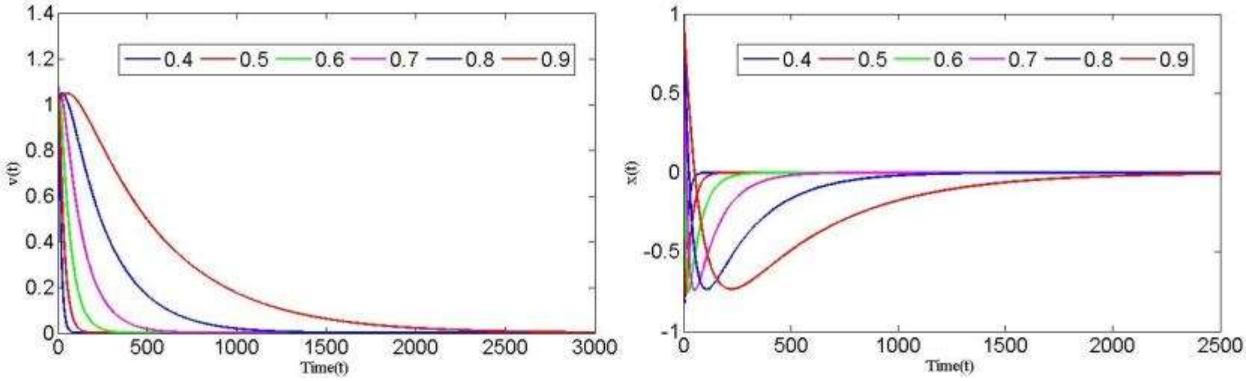

Figure 9: Overdamped oscillations of (5) for different values of ν

Example 6 *Let $R = 8.6, L = 7, C = 0.0238$ and $S = 0.001$ then the eigenvalues are $\lambda_{1,2} = -2.449$. The Critically damped oscillations of the system (5) shown in Figure-(10) similar to over damped oscillations approaches the equilibrium position with increase in time.*

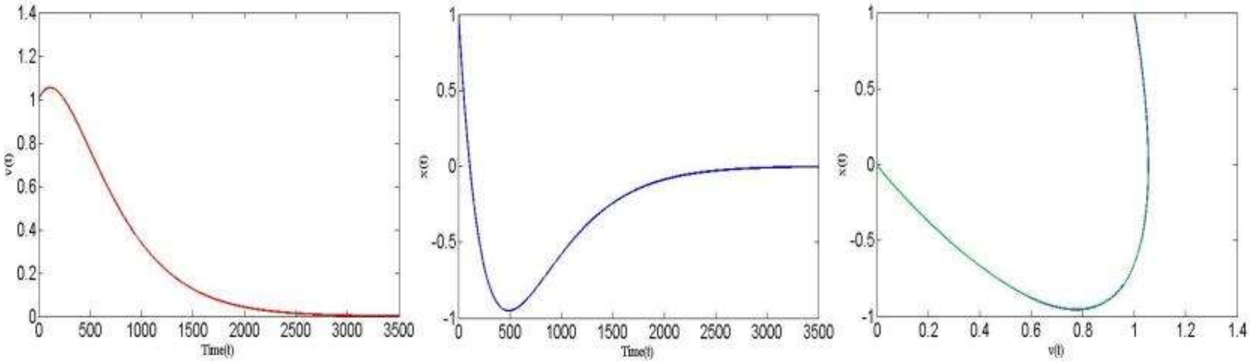

Figure 10: Critical Damping

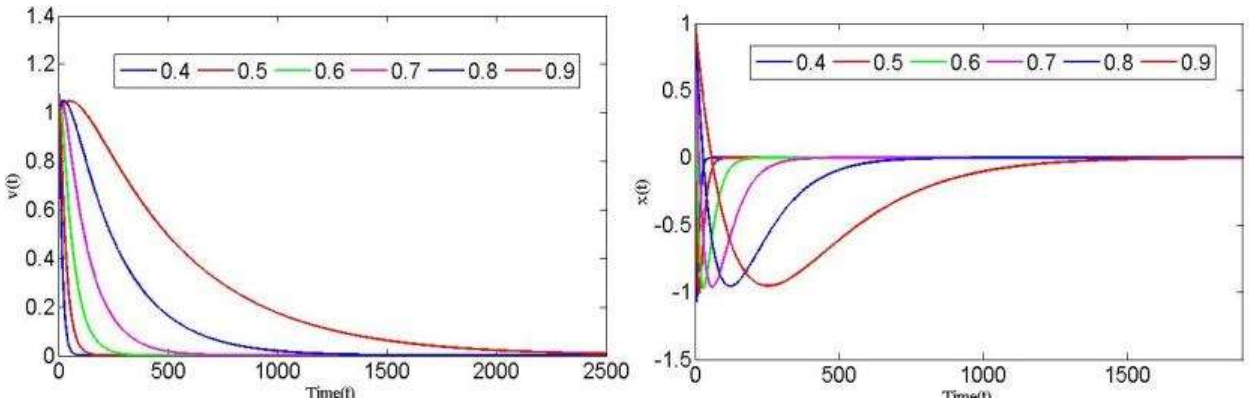

Figure 11: Critically damped oscillations of (5) for different values of ν

Example 7 *Consider the system (5) without the damping term* β*.The eigenvalues* $\lambda_{1,2} = 1 \pm i\ \omega_0$*, are clearly complex, conjugate of the form* $c \pm i\ d$ *with* $c > 0$*. Figure-(12) represents the Numerical simulations for values* $L = 2$ *and* $C = 0.01389$ *of the System (5). The phase portrait with outgoing spiral explains that the equilibrium points of the system (5) is unstable.*

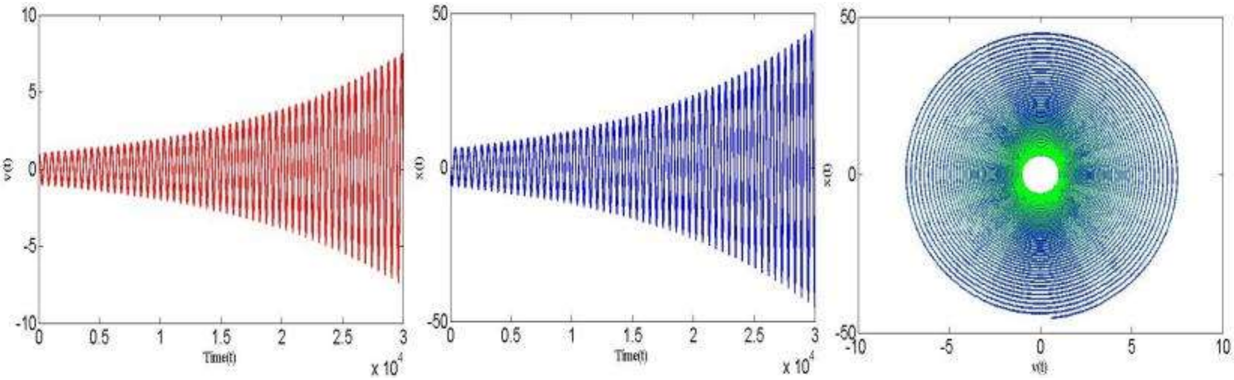

Figure 12: Un damped oscillations of (5)

IV CONCLUSION

The Stability properties of both continuous and Discrete Fractional 2-D model describing parallel RLC circuit is analyzed. Time plots and phase portraits are presented to show the Stability of the parallel RLC circuit.

REFERENCES

[1] R.Murali and A. Ponmana Selvan, “Ulam Stability of a LCR Electric Circuit with Electromotive Force”, Int. J. Math. And Appl., 6(1)(2018), (special Issue), 61-66.

[2] William H. Hayt, Jack E. Kemmerly and Steven M. Durbin, “Engineering Circuit Analysis”, Eighth Edition, McGraw-Hill Companies.Inc, 2012. ISBN 978-0-07-352957-8.

[3] Horowitz and Hill, “The Art of Electronics”, Second Edition, Cambridge University press, 1989. ISBN 0521370957.

[4] I. Petrás, “Fractional-Order Nonlinear Systems:Modeling, Analysis and Simulation”,Springer,2011.

[5] https://en.wikibooks.org/wiki/Circuit_Theory/All_Chapters

