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Generalized Fixed Point Theorem of pal and Maiti

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Abstract : *In this paper, we extend a unique fixed point of Pal and Maiti for any positive power of two self mappings in 2-metric space*

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Introduction: The notion of 2-metric was introduced by Gahler in 1963 as an abstract generalization of the concept of area function for Euclidean triangles. The concept of 2-metric attracted the attention of many researchers. Many authors like Iseki, Khan, Rhoades, Lal and Singh etc. probed deeply into this area and established several fixed point theorems in 2- metric space setting as generalizations or extensions to the metric fixed point theorems. Several fixed point theorems appeared in 2-metric spaces analogous to the fixed point theorems in metric space setting. In this present work we generalize the fixed point theorems that are proved by pal and maiti[4]. In 1977 Rhoades [6] proved some fixed point theorems by using contractive type mappings for 2-metric spaces.

1. Preliminaries

In this section, we present some basic definitions which are needed for the further study of this paper

1.1 Definition: Let (X,d) be a 2 –metric space. A mapping $T: X \to X$ is said to be Contractive if for all x,y,a in X

$$d(Tx, Ty, a) < d(x, y, a)$$

1.2 Definition: A 2-metric on a non-empty set X is a function $d : X^3 \longrightarrow R$, satisfying the following properties.

(a) $d(x, y, z) = 0$, if at least two of x,y,z are equal

(b) for each pair of distinct points x , y in X there exists a point $z \in X$ such that $d(x, y, z) \neq 0$

(c) $d(x, y, z) = d(x, z, y) = d(y, z, x)$ for all x, y, z in X

(d) $d(x, y, z) \leq d(x, y, u) + d(x, u, y) + d(u, y, z)$ for all x, y, z and u in X

then d is called a 2-metric on X and the pair (X ,d) is called a 2-metric space

1.3 Remark: A Contractive mapping of a complete 2 –metric space (X,d) into itself need not have a fixed point.

1.4 Example: Let $x = \{ x \in R : x \geq 1 \}$ with 2 –metric defined as

$$d(x,y,z) = \min \{ |x - y|, |y - z|, |z - x| \}$$

Let $F(x) = x + \frac{1}{x}$, then $F(1) = 2$, $F(2) = 2.5$, $F(3) = 3.33$ and so on.

d(F(1), F(2), F(3)) = d(2, 2.5, 3) = ½
d(1,2,3) = 1
But , ½ <1 , so F is a Contractive but it has no fixed point

2. Generalized fixed point theorem

2.1 Theorem: : Let (X,d) be a complete 2–metric space and S, T : X → X
Such that for all x, y, a in X and positive integers p,q, (p + q)

$d(S^p(x), T^q(y), a) < \max.\{ d(x, y, a), d(x, S^p(x), a), d(y, T^q(y), a)$
$\frac{1}{2}[d(x, T^q(y), a) + d(y, S^p(x), a)]$

Then S and T have a unique common fixed point

Proof : Let for any arbitrary point $x_0 \in X$, $\{ x_n \}$ be a Cauchy sequence defined as

$$X_{2n+1} = S^p x_{2n} \ , \quad X_{2n} = T^q x_{2n-1} \ , n = 0,1,2,\ldots\ldots\ldots$$

Then from given condition

$d_{2n} = d\{x_{2n}, x_{2n+1}, a) = d(S^p x_{2n}\ T^q x_{2n-1}, a)$
$< \max\{d(x_{2n-1}, x_{2n}, a), d(x_{2n}, S^p x_{2n}, a), d(x_{2n-1}, T^q x_{2n-1}, a)$
$1/2[d(x_{2n}, T^q x_{2n-1}, a) + d(x_{2n-1}, S^p x_{2n}, a)] \}$

i.e., $d(x_{2n}, x_{2n+1}, a) < d(x_{2n-1}, x_{2n}, a)$

$d_{2n+1} = d(x_{2n+1}, x_{2n+2}, a) = d(S^p x_{2n}, T^q x_{2n-1}, a)$
$< \max. d(x_{2n}, x_{2n+1}, a), d(x_{2n}, S^p x_{2n}, a), d(x_{2n+1}, T^q x_{2n-1}, a),$
$\frac{1}{2}[d(x_{2n}, T^q x_{2n-1}, a) + d(x_{2n+1}, S^p x_{2n}, a)] \}$

i.e. , $d(x_{2n+1}, x_{2n+2}, a) < d(x_{2n}, x_{2n+1}, a) < d(x_{2n-1}, x_{2n}, a), \ldots\ldots\ldots d(x_0, x_1, a)$

Thus $d_{2n+1} < d_{2n} < \ldots\ldots\ldots < d_0$. so the sequence $\{ d_{2n} \}$ is monotone decreasing and bounded also

Thus $d_{2n} \to 1$ as $n \to \infty$. As X is compact, there exists a cluster point u in $\{x_n\}$
And so there exists a subsequence $\{ x_{2n} \} \to u$ as $n \to \infty$.

Also $x_{2n+1} = S^p x_{2n} \to S^p u$ and

$x_{2n+2} = T^q x_{2n+1} = T^q S^p x_{2n} \to T^q S^p u$ when $n \to \infty$.

Thus we get

$$1 = \lim_{n\to\infty} d(x_{2n}, x_{2n+1}, a) = \lim_{n\to\infty} d(x_{2n}, S^p x_{2n}, a) = d(u, S^p u, a)$$

$$1 = \lim_{n\to\infty} d(x_{2n+1}, x_{2n+2}, a) = \lim_{n\to\infty} d(S^p x_{2n}, T^q x_{2n+1}, a)$$
$$= \lim_{n\to\infty} d\left(S^p x_{2n}, T^q S^p x_{2n}, a\right)$$
$$= d(S^p u, T^q S^p u, a)$$

Suppose that $u \neq S^p u$, then

$d(u, S^p u, a) < d(u, S^p u, x_{2n}) + d(u, x_{2n}, a) + d(x_{2n}, S^p u, a)$
$= d(u, S^p u, x_{2n}) + d(u, x_{2n}, a) + d(T^q x_{2n-1}, S^p u, a)$
$< d(u, S^p u, x_{2n}) + d(u, x_{2n}, a) + \max\{ d(u, x_{2n-1}, a), d(u, S^p u, a),$
$d(x_{2n-1}, T^q x_{2n-1}, a), \frac{1}{2}[d(u, T^q x_{2n-1}, a) + d(x_{2n-1}, S^p u, a)] \}$
$= d(u, S^p u, x_{2n}) + d(u, x_{2n}, a) + \max\{d(u, x_{2n-1}, a), d(u, S^p u, a),$
$d(x_{2n-1}, x_{2n}, a), \frac{1}{2}[d(u, x_{2n}, a) + d(x_{2n-1}, S^p u, a)] \}$

When $n \to \infty$

$d(u, S^p u, a) < d(u, S^p u, a)$, which is impossible.Thus $S^p u = u$.

Similarly we can show that $T^q u = u$.

Then u is the common fixed point of S^p and T^q.

Next we show that u is the only common fixed point of S^p and T^q.

If possible let $u^* \neq u$ is a another common fixed point of S^p and T^q.

Then $S^p(u) = T^q(u) = u^*$.

Hence $d(u,u^*,a) = d(S^p u, T^q u^*, a) < \max.\{ d(u, u^*, a), d(u, S^p u, a), d(u^*, T^q u^*, a),$

$$\frac{1}{2} [d(u, T^q u^*, a) + d(u^*, S^p u, a)] \}$$

$d(u,u^*,a) < d(u,u^*,a)$ which is a controdiction

Thus $u = u^*$.

Hence u is the unique common fixed point of S^p and T^q.

If we put $p = q = 1$ then u is a unique common fixed point of S and T.

Remark: If we put S = T and p = q = 1Then we get an analogee of pal and Maiti with condition (d) [4] in 2 –metric space.

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