OPTIMIZATION OF SPUR GEAR DESIGN FOR MINIMUM OIL CHURNING LOSS BASED ON GENETIC ALGORITHM

Janak K. Sangani, Dr. K. D. Maniya

Abstract: This paper presents work on the optimization method for spur gear design to minimize oil churning losses. Empirical model is used to study the oil churning losses; such model is validated using data available in literature. The oil churning losses appear if the rotating components are partially submerged in an oil bath. In this paper, Gear dimension such as module, pitch diameter, face width and oil churning power loss are optimized and compared with the experimental data in literature. The main objective function is to minimize oil churning power loss and constraints are minimum center distance, minimum gear modulus and minimum number of teeth, surface compressive stress and bending stress. A gear transmission problem is one of the most complex optimization problems because of relationship between different variables. Therefore researchers are now going to use Non-classical Techniques. Genetic Algorithm is one of such techniques.

Index Terms— Genetic Algorithm, Minimum oil churning loss, Empirical models, Spur gear

1 INTRODUCTION

The power losses of a gearbox containing several gear pairs that are supported by shafts and rolling element bearings can be classified into two groups. The first group is load-dependent (mechanical) power losses caused primarily due to contacting surfaces of gears and bearings. The losses in the second group are load-independent (spin) power losses. Load independent power loss are oil churning losses and air windage losses. Load dependent power loss are rolling losses and sliding losses.

In spin power losses are directly related to the lubrication method. Oil churning losses appear if the rotating components are partially submerged in an oil bath. Oil churning power loss is defined as the power loss when a gear is running in an oil bath or is dipping into oil “slugs”. Churning losses result from gear blanks revolving in the oil sump and generating splash for lubricating the gear teeth, bearings, and seals. Churning losses are a function of speed and oil level, oil viscosity, tooth geometry and submerged depth of the gears. Several empirical formulas were proposed to estimate the oil churning losses.

Conventional optimization methods have been widely used in various mechanical design problems. They are deterministic in nature and use only a few geometric design variables due to their complexity and convergence problems. When the number of design parameters increases, the complexity increases. If the optimization problem involves the objective function and constraints that are not stated as explicit functions of the design variables or which are too complicated to manipulate, it is hard to solve by classical optimization methods [1].

Hence it is necessary to develop more efficient and reliably technique that solves such problems. GA differs substantially from non-classical optimization methods.

Genetic algorithm is based on evolutionary processes and Darwin’s concept of natural selection. It works on the principle that, only the fittest populations will survive while the bad populations are weeded out.

2 LITERATURE REVIEW

The literature review has been classified based on power loss and gear design

Terekhov [2]. In a paper the author conducted numerous experiments with high viscosity lubricant (200-2000Cst), low rotational speeds and tested gear with a modulus ranging from 2 to 8 mm. The churning losses $P_{churning}$ are independent on the gear tooth geometry and dependent on the viscosity of the lubricant. Lauster and Boos [3] have developed dimensionless drag torque to their own measurement sets. Bones [4] discusses drag torque increases with an increasing Reynolds number. Changenet and vleux [5] this paper evaluates the minimize energy losses, one possible way to improve churning loss consists in changing the shape of the casing. Test spur gear with number of teeth 20 to 102, face widths of 14 and 24mm and modules of 1.5 to 5 mm. Seabra et al. [6] considered the churning power loss are dependent of gear immersed depth, rotating speed, lubricant viscosity and density, gearbox geometry.

Literature Review based on Gear design optimization

Mendi et al.[1] optimized volume of gearbox. The gear volume obtained by GA is 1.47% lower than the gear volume obtained by Analytical method. Yokota [7] used Genetic Algorithm to minimize the weight of the gear for a constrained bending strength of gear, surface durability, torsional strength for pinion and gear and centre distance while face width, diameter of pinion and gear shaft, no of teeth on pinion and gear module as design variables. Mohan and Seshiah [8] have introduced genetic algorithm to design multi objective optimized spur gear. Minimization of centre distance, minimization of weight and minimization of tooth deflection are taken as objective functions. Also module, face width & teeth
3 STATEMENT OF OPTIMIZATION PROBLEM

3.1 Design variables

Here pitch diameter (Dp), face width (b) and module (m) are considered as design variables.

F(x) = F(Dp, b, m) = F(X1, X2, X3)

Upper and lower bounds of design variables are,

0.09 ≤ Dp ≤ 0.212, 0.016 ≤ b ≤ 0.048, 0.002 ≤ m ≤ 0.004

3.2 Formulation of objective function

Churning losses depend mainly on the rotational speed, immersion depth, pitch diameter, gear width, lubricant density and oil volume [5].

Minimum oil churning loss

\[ P_{ch} = \frac{1}{2} \rho \Omega^3 R_p^3 S_m C_m \]  

(1)

Where,

\[ C_m = 1.366 \left( \frac{h}{D_p} \right)^{0.41} \left( \frac{V_c}{D_p} \right) \frac{Fr^{0.41} Re^{-0.21}}{b} \]  

(2)

\[ S_m = R_p^2 (2 \theta - \sin 2 \theta + D_p \theta + 2 (Z \theta H_{tooth} b)) / (\pi \cos \theta) \]  

(3)

Substituting of Eq. (2), (3) in Eq. (1)

\[ P_{ch} = \frac{1}{2} \rho \Omega^3 \left( \frac{D_p}{V_c} \right) \left( \frac{D_p}{b} \right) \left( \frac{h}{D_p} \right)^{0.41} \left( \frac{V_c}{D_p} \right) \frac{Fr^{0.41} Re^{-0.21}}{b} \]  

(4)

3.3 Formulation of design constraints

Design constraints are converted in terms of design variable like pitch diameter, face width and module. The constraints considered for the optimum design of spur gear is as follows [13]

1. Minimum centre distance:

\[ D_1 \left( \frac{1+i}{\pi} \right) \geq \left( i+1 \right) \left[ \frac{0.74}{\sigma_i} \right] E[M_i]^{0.8} \]  

2. Surface compressive stress:

Permissible surface compressive stress ≥ Surface compressive stress

\[ [\sigma_s] \geq \frac{1.48 \sqrt{2+i}}{D_p \cos \theta b} E[M_i] \]  

3. Minimum number of gear module:

Module ≥ Minimum number of gear module

\[ m \geq 1.26 \left[ \frac{M_i}{m} \right] \]  

4. Bending stress:

Permissible bending stress ≥ bending stress

\[ [\sigma_b] \geq \frac{2i}{D_p \cos \theta b} [M_i] \]  

5. Minimum number of teeth:

\[ \frac{D_P^2 (1+i)}{\pi m_i} \geq 2 \pi \frac{\tan \sigma}{\sin \sigma} \]  

4 CASE STUDY

Optimize spur gear design for minimum oil churning losses for the following case study in table 1.

For all the cases

- Pitch diameter = 0.159m, Face width = 0.024m, Module = 0.003m
- Velocity ratio i = 2

Table 1: given data for different case study [5].

<table>
<thead>
<tr>
<th>Case study</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input power</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Rotational speed</td>
<td>1000</td>
<td>2000</td>
<td>3000</td>
</tr>
<tr>
<td>Torque transmitted (N.m)</td>
<td>18.63</td>
<td>9.308</td>
<td>6.205</td>
</tr>
<tr>
<td>Experiment Power loss (w)</td>
<td>17</td>
<td>51</td>
<td>97</td>
</tr>
</tbody>
</table>

The parameters considered for the spur gear design is given below.

Material of gear = carbon steel (C-45)

Permissible bending stress \([\sigma_b] = 67.08 \times 10^6 \text{ N/m}^2\)

Surface compressive stress \([\sigma_s] = 142.93 \times 10^6 \text{ N/m}^2\)

Young’s modulus of the material \((E) = 2.108 \times 10^6 \text{ N/m}^2\)

Pressure angle \((\sigma) = 20^\circ\)

Product of load concentration factor (k) and dynamic load factor \((k_d) = 1.3\)

Design torque transmitted \([M_i] = 18.61 \text{ N.m (case-1)}\)

Lewis form factor for spur gear \((Y) = 0.410\)

Lubricant properties [5]:
Kinematic viscosity $\nu = 48 \times 10^{-6}$ m$^2$/s, density $\rho = 873$ kg/m$^3$

Finally optimization problem for the above data is written as, (case-1)

Find out design variables

$$F(x) = F(D_p, b, m) = F(X_1, X_2, X_3)$$

Which minimize

$$P_{ch}(x) = \text{oil churning power loss}$$

$$P_{ch} = 253.50 \times 10^2 (X_1^{0.132} X_2^{0.7227} X_3 + 58.37 X_2 X_3)$$

Subject to design constraint

$$g_1 = X_1 - 53 X_3$$
$$g_2 = 18.10 \times X_1 \geq 1$$
$$g_3 = X_1 X_2^{0.5} \geq 5.026 \times 10^{-3}$$
$$g_4 = X_1^{0.33} X_2^{0.66} \geq 6.471 \times 10^{-3}$$
$$g_5 = X_1 X_2 X_3 \geq 2.709 \times 10^{-6}$$
$$g_6 = X_1 X_3^3 \geq 11.50$$

Similarly objective function and constraint is developed for case number 2 and 3.

5 IMPLEMENTATION OF GENETIC ALGORITHM

5.1 Fitness Function

To use Genetic Algorithm Toolbox functions and writes an M-file in computer the function to optimize. Specify the function as a function handle of the form @simplefitness, where simplefitness.m is an M-file returns a scalar is shown below.

Function $y = \text{simplefitness}(x)$

$$y = (253.50 \times 10^2) (x(1)^{0.189})(x(2)^{0.21}) (0.1132 \times (x(1)^2) + 0.7227 \times (x(2)^2) + 58.379 \times (x(2))^2)$$

5.2 Bounds

Specify lower bounds as [0.09, 0.016, 0.002] and upper bounds as [0.212, 0.048, 0.004]

5.3 Nonlinear Constraint Function

It defines the nonlinear constraints. Specify the function as an anonymous function or as a function handle of the form @simpleconstraint, where simpleconstraint.m is an M-file that returns the vectors c and ceq as shown below. The nonlinear equalities are of the form $ceq = 0$, and the nonlinear inequalities are of the form $c \leq 0$.

Function $[c, ceq] = \text{simpleconstraint}(x)$

$$c = 1 - 18.10 \times (x(1)) (5.026 \times (10^{-3} - (x(1)) (x(2))^{0.5})); (2.709 \times (10^{-6} - (x(1)) (x(2))^{2} (x(3)))); (6.471 \times (10^{-3}) - (x(1)) (x(2))^{2} (x(3)));$$

$$ceq = (x(1) - (x(3))^{33})$$

5.4 Different parameters of GA

Different parameters and its selected value are shown in Table 2.

Table 2: Selected Values of Different Parameters for GA

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>20</td>
</tr>
<tr>
<td>Initial Range</td>
<td>[0;1]</td>
</tr>
<tr>
<td>Elite Count</td>
<td>2</td>
</tr>
<tr>
<td>Crossover Fraction</td>
<td>0.8</td>
</tr>
<tr>
<td>Generation</td>
<td>50</td>
</tr>
</tbody>
</table>

6 RESULTS AND DISCUSSION

The figure 1 shows the optimized values of design variables. It plots the vector entries of the individual with the best fitness function value in each generation.(case-1)

Figure 1: Current Best Individual vs. Number of Variables

Same plots for case number 2 and 3 but different value of power loss

Actual and GA values of design variable and objective function are tabulated below in table 3.

Table 3: Comparison of values of Design Variables & Objective Functions, those are obtained actually and using Genetic Algorithm.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Actual [5]</th>
<th>Values of GA</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>159</td>
<td>106</td>
<td>106</td>
</tr>
<tr>
<td>Diameter (mm)</td>
<td>159</td>
<td>107</td>
<td>107</td>
</tr>
<tr>
<td>Face width (mm)</td>
<td>24</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Module (mm)</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Churning loss (w)</td>
<td>17</td>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

Above table shows that, there is 33.33% reduction in pitch diameter, face width, module and 75% reduction in oil churning power loss.

7 CONCLUSION

In this study the attempt has made to optimize oil churning loss with different constraint such as surface compressive stress, bending stress, minimum number of teeth and minimum number of gear module. The variable considered are pitch diameter, face width and module. Result shows that, Gear dimension such as module, pitch diameter, face width and oil churning power loss are optimized and compared with the experimental data in literature and all objectives are satisfied. So GA is one of the best tool for optimization problem.
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REFERENCES