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Balanced V_4 - Cordial graph and its application to Produce new V_4 - Cordial Families

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Abstract : *In this paper we introduce a balanced V_4 - cordial labeling for a graph G. We proved that $P_n \times C_{8t}$, $C_n \times C_{8t}$, are balanced V_4 - cordial. We also proved that the corona graph $G_1 \odot G_2$ is V_4 - cordial, when G_1 is V_4 - cordial and G_2 is balanced V_4 - cordial.*

Keywords :*Binary vertex labeling, balanced cordial graph, corona graph.*
AMS Subject Classification number: 05C78.

1 Introduction

Gallian [3] survey provides vast amount of literature with bibiliography of papers on different types of graph labeling. Among some papers show labeled graphs have many diversified applications. The cordial labeling introduced by Cahit[1] is a weaker version of graceful and harmonious labeling. After this many researchers have studied cordial graphs and similar type graph labeling. Among one of these is V_4 - cordial labeling which was introduced by Riskin [5] in 2013.

We follow Harary [2] for the basic notation and terminology of graph theory. Let $Z_2 \times Z_2 = \{0 = \langle 0,0 \rangle, a = \langle 1,0 \rangle, b = \langle 0,1 \rangle, c = \langle 1,1 \rangle\} = V_4$ be the Klein-four group with the binary operation $*$. The graph G with vertex set V(G) and edge set E(G) is said to be V_4 - cordial if there exist a mapping $f: V(G) \longrightarrow \{0, a, b, c\}$ which satisfies followings when the edge $e = (u, v) \in E(G)$ labeled by $f(u) * f(v)$.

1. $|v_f(p) - v_f(q)| \le 1, \forall p, q \in V_4$
2. $|e_f(p) - e_f(q)| \le 1, \forall p, q \in V_4$

Where,
$v_f(p)$ = the number of vertices in G with label p
$v_f(q)$ = the number of vertices in G with label q
$e_f(p)$ = the number of edges in G with label p and
$e_f(q)$ = the number of edges in G with label q.

Riskin [5] proved that K_n is V_4 - cordial iff $n \le 3$ and the star graph $K_{1,n}$is V_4 - cordial. Seenivasan and Lourdusamy [6] proved that an Eulerian graph G with $|V(G)| \equiv 2 (\text{mod } 4)$ has no V_4 - cordial labeling, all trees except P_4, P_5 are V_4 - cordial and cycle C_n is V_4 - cordial iff $n \in N - \{4,5\}$ and $n \equiv 2 (\text{mod } 4)$. Pechenik and Wise [4] proved that path P_n is V_4 - cordial unless $n \in \{4,5\}$, all ladders $P_2 \times P_k, k \ge 3$ are V_4 - cordial and the d - dimensional hypercube $Q_d (d \ge 3)$ is V_4 - cordial.

A V_4 - cordial graph G with a V_4 - cordial labeling f is a balanced V_4 - cordial graph if $v_f(0) = v_f(a) = v_f(b) = v_f(c) = |V(G)|/4$ and $e_f(0) = e_f(a) = e_f(b) = e_f(c) = |E(G)|/4$. It is said to be edge balanced V_4- cordial if $|e_f(p) - e_f(q)| = 0, |v_f(p) - v_f(q)| \le 1, \forall p, q \in V_4$ and

$\left|v_f(p) - v_f(q)\right| = 1$, for some $p, q \in V_4$. A V_4 - cordial graph G is said to be vertex balanced V_4 - cordial if $\left|v_f(p) - v_f(q)\right| = 0, \left|e_f(p) - e_f(q)\right| \leq 1\ \forall p, q \in V_4$ and $\left|e_f(p) - e_f(q)\right| = 1$, for some $p, q \in V_4$. Similarly it is said to be unbalanced V_4 - cordial graph if $\left|v_f(p) - v_f(q)\right|, \left|e_f(p) - e_f(q)\right| \in \{0,1\}, \forall p, q \in V_4$ and $\left|v_f(p) - v_f(q)\right|, \left|e_f(p) - e_f(q)\right| \in \{0,1\}, for\ some\ p, q \in V_4$.

For any V_4 - cordial graph G, if f is a V_4 - cordial labeling for G and it has one of the above four categories, then g is also a V_4 - cordial labeling function for G and it has same one of the above four categories, where g: V(G) → V_4 defined by f(u)=g(u), when f(u)=0 and { f(x), f(y), f(z) } = { g(x), g(y), g(z)} when f(x),f(y),f(z) ∉ {0}.

The corona graph $G_1 \odot G_2$ is obtained from two graphs G_1 and G_2, by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and join the i^{th} vertex of G_1 with all the vertices in the i^{th} copy $G_2^{(i)}$ of G_2 by an edge.

2 Main Results

Theorem - 2.1

$P_n \times C_{8t}$ is a balanced V_4 - cordial graph, where $t, n \in \mathbb{N}$.

Proof: Let $u_{i,j}$ $(1 \leq j \leq 8t)$ be the vertices of i^{th} copy $C_{8t}^{(i)}$ in $P_n \times C_{8t}, \forall\ i = 1,2, \ldots, n$. The vertex labeling function $f_1: V(C_{8t}^1) \rightarrow \{0, a, b, c\}$ defined by

$$f_1(u_{1,j}) = \begin{cases} 0\ if\ j \equiv 1,5 (mod\ 8) \\ a\ if\ j \equiv 2,3 (mod\ 8) \\ b\ if\ j \equiv 6,7 (mod\ 8) \\ c\ if\ j \equiv 0,4 (mod\ 8) \end{cases} \forall j = 1,2, \ldots 8t.$$

is a balanced V_4 - cordial labeling for $C_{8t}^{(1)}$. For each i=1,2,...,n-1, we join $u_{i,j}$ with $u_{i+1,j}$, ∀ j=1,2,...,8t by an edge to form the graph $P_n \times C_{8t}$. Define $f_2: V(C_{8t}^2) \rightarrow \{0, a, b, c\}$ as follows.

$$f_2(u_{2,j}) = \begin{cases} 0\ if\ j \equiv 1,5 (mod\ 8) \\ a\ if\ j \equiv 0,4 (mod\ 8) \\ b\ if\ j \equiv 2,3 (mod\ 8) \\ c\ if\ j \equiv 6,7 (mod\ 8) \end{cases} \forall j = 1,2, \ldots 8t.$$

Note that above defined labeling function f_2 on C_{8t}^2 is also a balanced V_4 - cordial labeling. Now defined $f(P_n \times C_{8t}) \rightarrow V_4$ as follows,

$f(u_{i,j}) = f_1(u_{1,j})$ if i is odd
$= f_2(u_{2,j})$ if i is even ∀ i=1,2,...,n, ∀j=1,2,...,8t.

For each i=1,2,...,n and j=1,2,...,8t, it is observe that

$$f(u_{i,j}) * f(u_{i,j+1}) = \begin{cases} 0, if\ j \equiv 1,5 (mod\ 8) \\ a, if\ j \equiv 6,7 (mod\ 8) \\ b, if\ j \equiv 0,4 (mod\ 8) \\ c, if\ j \equiv 2,3 (mod\ 8) \end{cases}$$

Thus, $|e_f(p) - e_f(q)| = 0, \forall p, q \in V_4$ in $P_n \times C_{8t}$. Since $V(P_n \times C_{8t}) = \cup_{i=1}^{n} V(C_{8t}^{(i)})$, it is observe that $|v_f(p) - v_f(q)| = 0, \forall p, q \in V_4$ in $P_n \times C_{8t}$, as f_1, f_2 both are balanced V_4 - cordial labeling on C_{8t}. Hence, $P_n \times C_{8t}$ is a balanced V_4 - cordial graph.

Theorem - 2.2
$C_n \times C_{8t}$ and its balanced V_4 - cordial labeling, when $n \leq 3$.

Proof: Let $u_{i,j}$ $(1 \leq j \leq 8t)$ be the vertices of i[th] copy $C_{8t}^{(i)}$ in $C_n \times C_{8t}, \forall\, i = 1,2, \ldots, n$. The vertex labeling function $f_1: V(C_{8t}^1) \to \{0, a, b, c\}$ defined by

$$f_1(u_{1,j}) = \begin{cases} 0 \; if \; j \equiv 1,5(mod\; 8) \\ a \; if \; j \equiv 2,3(mod\; 8) \\ b \; if \; j \equiv 6,7(mod\; 8) \\ c \; if \; j \equiv 0,4(mod\; 8) \end{cases} \forall j = 1,2, \ldots 8t.$$

is a balanced V_4 - cordial labeling for $C_{8t}^{(1)}$. For each i=1,2,...,n-1, ∀ j=1,2,...,8t, we join we join $u_{i,j}$ with $u_{i+1,j}$ and $u_{8t,j}$ with $u_{1,j}$ by an edge to form the graph $C_n \times C_{8t}$. Define $f_2: V(C_{8t}^2) \to \{0, a, b, c\}$ as follows.

$$f_2(u_{2,j}) = \begin{cases} 0 \; if \; j \equiv 1,5(mod\; 8) \\ a \; if \; j \equiv 0,4(mod\; 8) \\ b \; if \; j \equiv 2,3(mod\; 8) \\ c \; if \; j \equiv 6,7(mod\; 8) \end{cases} \forall j = 1,2, \ldots 8t.$$

Define $f_2: V(C_{8t}^n) \to \{0, a, b, c\}$ as follows.

$$f_3(u_{n,j}) = \begin{cases} 0 \; if \; j \equiv 1,5(mod\; 8) \\ a \; if \; j \equiv 6,7(mod\; 8) \\ b \; if \; j \equiv 0,4(mod\; 8) \\ c \; if \; j \equiv 2,3(mod\; 8) \end{cases} \forall j = 1,2, \ldots 8t.$$

Note that above defined labeling functions f_2 on $C_{8t}^{(2)}$ and f_3 on $C_{8t}^{(n)}$ both are balanced V_4 - cordial labeling.

Now defined $f: (C_n \times C_{8t}) \to$ V4 as follows

$$f(u_{n.j}) = f_3(u_{n,j}) \forall \text{ j=1,2,...,8t}$$

$$f(u_{i,j}) = \begin{cases} f_1(u_{1,j}) \; if \; i \; is \; odd \\ f_2(u_{2,j}) \; if \; i \; is \; even, \end{cases} \forall i = 1,2, \ldots, n-1, \forall j = 1,2, \ldots, 8t.$$

For each $i = 1,2, \ldots, n-2, j = 1,2, \ldots, 8t$, it is observe that

$$f(u_{i,j}) * f(u_{i+1,j}) = \begin{cases} 0, if\ j\ \equiv 1{,}5 (mod\ 8) \\ a, if\ j\ \equiv 6{,}7 (mod\ 8) \\ b, if\ j\ \equiv 0{,}4 (mod\ 8) \\ c, if\ j\ \equiv 2{,}3 (mod\ 8) \end{cases}$$

$$f(u_{n-1,j}) * f(u_{n,j}) = \begin{cases} 0, if\ j\ \equiv 1{,}5 (mod\ 8) \\ a, if\ j\ \equiv 0{,}4 (mod\ 8) \\ b, if\ j\ \equiv 2{,}3 (mod\ 8) \\ c, if\ j\ \equiv 6{,}7 (mod\ 8) \end{cases}$$

$$f(u_{n,j}) * f(u_{1,j}) = \begin{cases} f(u_{n-1,j}) * f(u_{n,j}), when\ n\ is\ even \\ 0, when\ n\ is\ odd\ and\ if\ j \equiv 1{,}5 (mod\ 8) \\ a, when\ n\ is\ odd\ and\ if\ j \equiv 2{,}3 (mod\ 8) \\ b, when\ n\ is\ odd\ and\ if\ j \equiv 6{,}7 (mod\ 8) \\ c, when\ n\ is\ odd\ and\ if\ j \equiv 0{,}4 (mod\ 8) \end{cases}$$

Thus, $|e_f(p) - e_f(q)| = 0$, in $C_n \times C_{8t}, \forall p, q \in V_4$. Since $(C_n \times C_{8t}) = \cup_{i=1}^{n} V(C_{8t}^{(i)})$, it is observe that $|v_f(p) - v_f(q)| = 0, \forall p, q \in V_4$ in $C_n \times C_{8t}$, as $f_i (i = 1,2,3)$ are balanced V_4 - cordial labeling on C_{8t}. Hence, $C_n \times C_{8t}$ is a balanced V_4 - cordial, $\forall n \geq 3$.

Theorem - 2.3
The corona graph $G_1 \odot G_2$ is V_4 - cordial, when G_1 is V_4 - cordial and G_2 is balanced V_4 - cordial.

Proof: Let $G_1 \odot G_2$ be the corona graph obtained by two V_4 - cordial graphs G_1 and G_2, among G_2 is V_4 - cordial. Let $|V(G_1)| = p_1, |V(G_2)| = p_2, f_1$ be a V_4- cordial labeling function for G_1 and f_2 be a balanced V_4 - cordial labeling function for G_2. It is obvious that $|e_{f_1}(p) - e_{f_1}(q)| \leq 1, |v_{f_1}(p) - v_{f_1}(q)| \leq 1, \forall p, q \in V_4$ in G_1 and $e_{f_2}(p) - e_{f_2}(q) = v_{f_2}(p) - v_{f_2}(q) = 0, \forall p, q \in V_4$ in G_2.

Let $G_1 \odot G_2$ Note that $|V(G)| = p_1(p_2 + 1)$ and $|E(G)| = p_1(p_2 + |E(G_2)|) + |E(G_1)|$. Now define $f: V(G) \to \{0, a, b, c\}$ as follows.

$$f(x) = \begin{cases} f_1(x), if\ x \in V(G_1) \\ f_2(x), if\ x \in V(G_2^i), \end{cases} \forall i = 1,2, \ldots p_1.$$

Let $V(G_1) = \{u_1, u_2, \ldots, u_p\}$ be the vertex set for G_1. For each $i = 1,2, \ldots, p_1$ join u_i with all the vertices of $G_2^{(i)}$ by an edge to form the corona graph $G = G_1 \odot G_2$. Note that each $u_i \in V(G_1)$ and i[th] copy $G_2^{(i)}$ produce p_2 edges, among quarter edges got 0 edge label, equal number of edges got a, b, c edge labels, as $v_{f_2}(0) = v_{f_2}(a) = v_{f_2}(b) = v_{f_2}(c) = p_2/4$ Therefore,

$$e_f(0) = p_1 e_{f_2}(0) + e_{f_1}(0) + p_1 p_2/4,$$
$$e_f(a) = p_1 e_{f_2}(a) + e_{f_1}(a) + p_1 p_2/4,$$
$$e_f(b) = p_1 e_{f_2}(b) + e_{f_1}(b) + p_1 p_2/4,$$
$$e_f(c) = p_1 e_{f_2}(c) + e_{f_1}(c) + p_1 p_2/4,$$
$$v_f(a) = v_{f_1}(a) + p_1 v_{f_2}(a),$$
$$v_f(b) = v_{f_1}(b) + p_1 v_{f_2}(b),$$
$$v_f(c) = v_{f_1}(c) + p_1 v_{f_2}(c), \text{ and}$$
$$v_f(0) = v_{f_1}(0) + p_1 v_{f_2}(0).$$

Thus,
$|e_f(p) - e_f(q)| = |e_{f_1}(p) - e_{f_2}(q)| \leq 1, p, q \in V_4$ and
$|v_f(p) - v_f(q)| = |v_{f_1}(p) - v_{f_2}(q)| \leq 1, p, q \in V_4$ as G_1 is V_4 - cordial. So, above defined labeling pattern give rise V_4 - cordial labeling to the corona graph $G_1 \odot G_2$. Therefore, $G_1 \odot G_2$ is V_4 - cordial.

Corollary - 2.4
Corona graph $C_{8m} \odot C_{8n}$ is balanced V_4 - cordial.

Corollary - 2.5
Corona graphs $P_{8m} \odot P_{8n}$ is vertex balanced V_4 - cordial.

Corollary - 2.6
Corona graph $P_{8m+1} \odot C_{8n}$ is a edge balanced V_4 - cordial.

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