Scientific Journal of Impact Factor (SJIF): 4.14

e-ISSN : 2348-4470
p-ISSN : 2348-6406

International Journal of Advance Engineering and Research Development

Volume 3,Issue 4,April -2016

Solution of counter-current imbibition Phenomena arising in Fluid Flow Through Porous Media with Inclination and Gravitational Effect using Homotopy analysis method

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Abstract — In this paper, counter- current Imbibition phenomenon in homogeneous porous media with inclination effect is discussed. Homotopy analysis method is used here to study the saturation rate for homogeneous porous media. A detailed discussion of saturation rate at various inclination angle has been done with dimensionless time and a comparison study with its physical interpretation has been done by using mathematica software

Keywords*-counter-current, imbibition, gravitation, homotopy analysis method, porous media.*

I. INTRODUCTION

In co-current flow, water and oil flow in the same direction, and water pushes oil out of the matrix. In counter-current flow, oil and water flow in opposite directions and oil escapes by flowing back in the same direction along which water has imbibed.

Many researchers studied this problem with different approaches. Scheidegger [1] described physics of oil-water motion in porous medium. Verma [4] [3] employed perturbation procedure and similarity methods to obtain explicit form analytical solutions of the imbibition equation in a heterogeneous cracked porous medium. Mehta and Patel [7] discussed analytically the phenomenon of imbibition in heterogeneous porous media during secondary oil recovery process. Pradhan and Verma [11] obtained the numerical solution of a specific imbibition phenomenon using Crank-Nicolson Scheme for finite differences. Meher et al. [8] examined the imbibition phenomena with capillary pressure and used the exponential self similar solution technique for finding the saturation rate in imbibition phenomena. Parikh [2] used the generalized separable solution of counter-current imbibition phenomena. Gohil and Meher [12] have used homotopy analysis method for solving Counter current imbibition phenomena of the time fractional type arising in the heterogeneous porous media.

In this paper, the phenomenon has been extended to homogeneous porous media with gravitational and inclination effect and used Homotopy analysis method to find an approximate analytical solution to the problem

II. STATEMENT OF THE PROBLEM

Here a cylindrical piece of homogeneous porous matrix of length L having its three sides are surrounded by an impermeable surface is considered whose one open end is labeled as imbibition face $x = 0$ and it is inclined at an angle α the ground surface. A Schematic diagram of this phenomenon is shown in fig. (1).

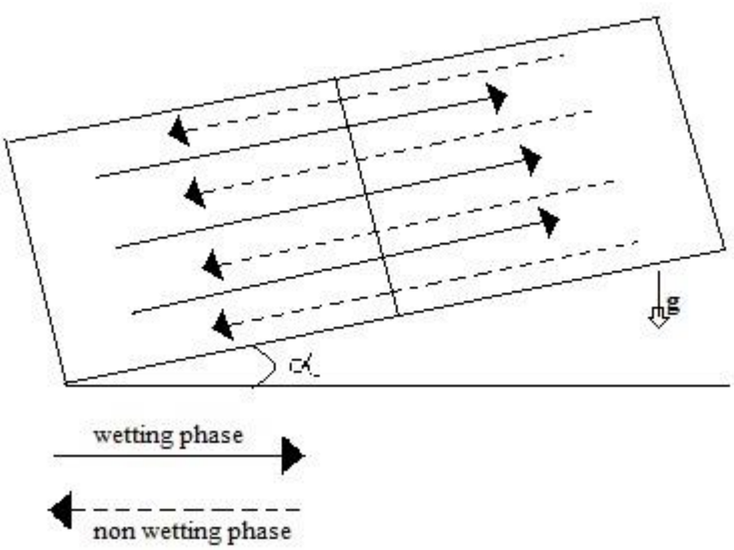

Figure 1: Schematic diagram of the problem under consideration

During imbibitions, when water is injected into an oil saturated porous matrix at imbibition face $x = 0$. The oil is displaced through a small distance $x = l$ due to the difference in phase viscosity as shown in fig. 1. Since water is injected at common interface in inclined homogeneous porous matrix contenting oil that will displace by injecting water.

III. MATHEMATICAL FORMULATION

Hence the water and oil both will satisfy Darcy's velocities of water and oil respectively as

$$v_w = -\frac{K_w}{\mu_w} K \left(\frac{\partial P_w}{\partial x} + \rho_w g \sin\alpha \right) \tag{1}$$

$$v_o = -\frac{K_o}{\mu_o} K \left(\frac{\partial P_o}{\partial x} + \rho_o g \sin\alpha \right) \tag{2}$$

Where K is permeability of homogeneous medium, K_o and K_w are relative permeability of oil and water which are function of S_o and S_w and ρ_w are constant densities of water and oil respectively, while μ_w and μ_o are constant kinematic viscosity of the phases in homogeneous porous media, α is the inclination of the bed, g is acceleration due to gravity. The coordinate x is measured along the axis of the cylindrical medium, the origin being located at the imbibition face $x = 0$. The flow is counter-current so for the imbibition phenomenon

$$v_w = -v_o \tag{3}$$

Therefore from equation (1) and (2), we may write

$$\frac{K_w}{\mu_w} K \left(\frac{\partial P_w}{\partial x} + \rho_w g \sin\alpha \right) + \frac{K_o}{\mu_o} K \left(\frac{\partial P_o}{\partial x} + \rho_o g \sin\alpha \right) = 0 \tag{4}$$

The definition of capillary pressure

$$p_c = p_o - p_w \tag{5}$$

Combining equation (4) and (5), we get

$$\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o} \right) \frac{\partial P_w}{\partial x} + g \sin\alpha \left(\rho_w \frac{K_w}{\mu_w} + \rho_o \frac{K_o}{\mu_o} \right) = 0 \tag{6}$$

Substituting the value of $\frac{\partial P_w}{\partial x}$ from the equation (6) into the equation (1), we obtain

$$v_w = \frac{K \left(\frac{K_w}{\mu_w} \right) \left(\frac{K_o}{\mu_o} \right) \left(\frac{\partial P_c}{\partial x} + (\rho_o - \rho_w) g \sin\alpha \right)}{\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o} \right)} \tag{7}$$

Since water and oil are following in a porous matrix through interconnected capillaries during the phenomenon of instability, due to capillary pressure of water and oil. Injected water and displaced native oil that will satisfy the equation of continuity as

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial v_w}{\partial x} = 0 \tag{8}$$

Where ϕ is the porosity of medium and S_w is the saturation of wetting phase. Eq. (8) with the value of v_w from eq. (7), becomes

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left[K \frac{K_w K_o \left(\frac{\partial P_c}{\partial x} + (\rho_o - \rho_w) g \sin\alpha \right)}{K_w \mu_o + K_o \mu_w} \right] = 0 \tag{9}$$

Eq. (9) is a non-linear partial differential equation, which describe the linear counter- current imbibition phenomenon of two immiscible fluid flow through homogeneous porous cylindrical medium with impervious boundary surface on three sides.

Analytical relationship between the relative permeability phase saturation and capillary pressure phase saturation as [5]

$$k_w = S_w$$
$$p_c = -\beta S_w \quad (10)$$

where β is constant of proportionality.

Since the present investigation involves water and viscous oil, therefore according to schidegger [1] we have

$$\left[\frac{K_w K_o}{K_w \mu_o + K_o \mu_w}\right] \approx \frac{K_w}{\mu_w} = \frac{S_w}{\mu_w} \quad (11)$$

Substituting the values of k_w and p_c from eq. (10) and eq. (11) into eq. (9), we get

$$\frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left[S_w \frac{\partial S_w}{\partial x} \frac{(\rho_o - \rho_w) g \sin\alpha}{\beta} S_w\right] = 0 \quad (12)$$

Now by using the dimensionless form

$$X = \frac{x}{L}, T = \frac{K\beta}{\mu\phi L^2} t$$

We get

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X}\left[S_w \frac{\partial S_w}{\partial X}\right] + \frac{(\rho_o - \rho_w) g \sin\alpha L}{\beta} \frac{\partial S_w}{\partial X} = 0 \quad (13)$$

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X}\left[S_w \frac{\partial S_w}{\partial X}\right] + g \sin\alpha D \frac{\partial S_w}{\partial X} = 0 \quad (14)$$

Where $D = \frac{(\rho_o - \rho_w) L}{\beta}$

With suitable initial condition $S_w(X,0) = e^{-X}$.

IV. RESULTS AND DISCUSSION

After applying homotopy analysis method on the governing equation (14), we get analytical solution $S_w(X,T)$. Using Mathematica software; all numerical values have been derived. In this calculation, we have taken $g = 9.8 m/s^2$. Results of Tables-1 to 4 and figure-2 to 5 shows when inclination increase $\alpha = 0^o$ to $\alpha = 15^o$ saturation slightly decrease. Also with time saturation rate also increased.

Table-1: Saturation V Time with inclination effect when X=0.2

T	$\alpha = 0^o$	$\alpha = 5^o$	$\alpha = 10^o$	$\alpha = 15^o$
0.2	0.824093	0.824010	0.823933	0.823866
0.4	0.829456	0.829290	0.829135	0.829052
0.6	0.834818	0.834569	0.834337	0.834273
0.8	0.840181	0.839849	0.839539	0.839273

Table-2: Saturation V Time with inclination effect when X=0.4

T	$\alpha = 0^o$	$\alpha = 5^o$	$\alpha = 10^o$	$\alpha = 15^o$
0.2	0.673915	0.673847	0.673783	0.673729
0.4	0.677509	0.677373	0.677247	0.677138
0.6	0.681104	0.680901	0.680710	0.684547
0.8	0.684699	0.684427	0.684173	0.686953

Table-3: Saturation V Time with inclination effect when X=0.6

T	$\alpha = 0^o$	$\alpha = 5^o$	$\alpha = 10^o$	$\alpha = 15^o$
0.2	0.551221	0.551166	0.551114	0.551069
0.4	0.553631	0.553519	0.553416	0.553326

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0.6	0.556040	0.555873	0.555718	0.555384
0.8	0.558456	0.558227	0.558020	0.557841

Table-4: Saturation V Time with inclination effect when X=0.8

T	$\alpha = 0^o$	$\alpha = 5^o$	$\alpha = 10^o$	$\alpha = 15^o$
0.2	0.450944	0.450899	0.450856	0.450820
0.4	0.452559	0.452468	0.452383	0.452231
0.6	0.454174	0.454038	0.453910	0.453801
0.8	0.455790	0.455607	0.455437	0.455291

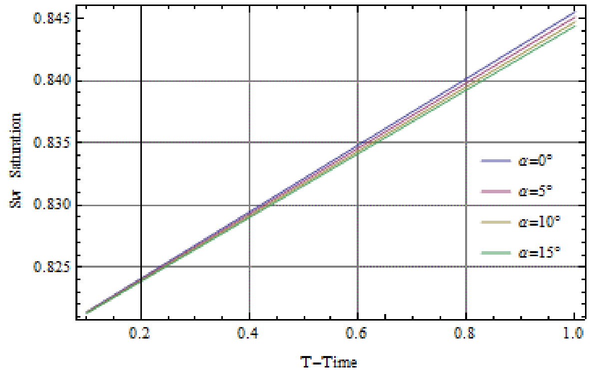

Figure-2 Saturation V Time [X=0.2]

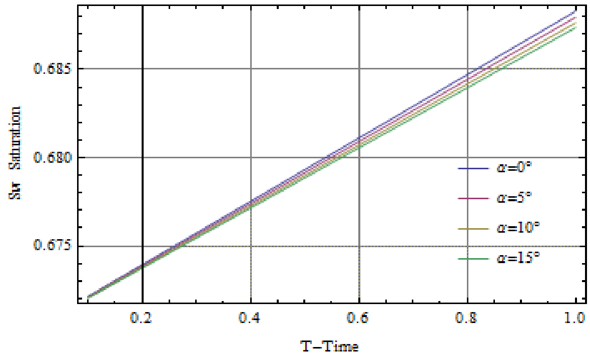

Figure-3 Saturation V Time [X=0.4]

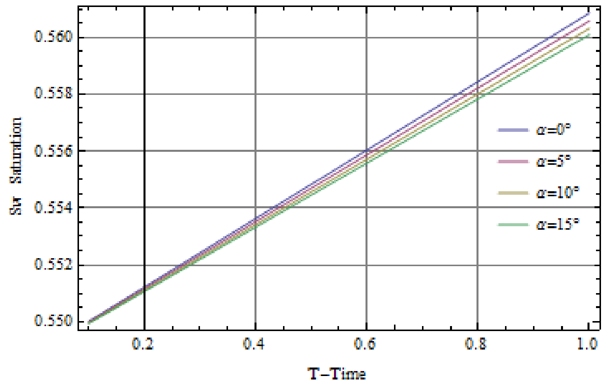

Figure-3 Saturation V Time [X=0.6]

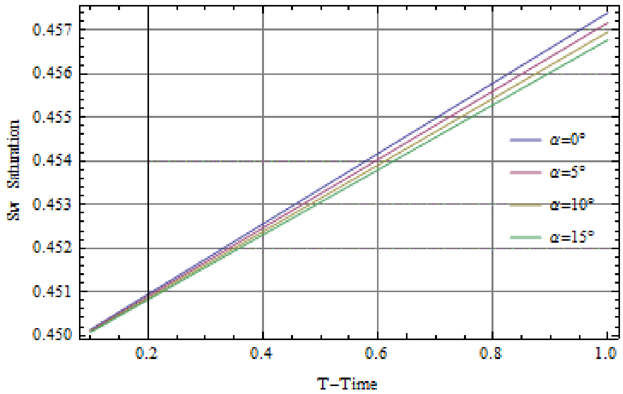

Figure-2 Saturation V Time [X=0.8]

IV. CONCLUSION

Homotopy analysis method for the case of counter-current imbibition phenomena has been applied successfully using freedom of choosing parameter h. Studied the variation of saturation of water in X and T direction for particular parametric values of the parameter α Equation (14) represents the saturation of wetting phase for counter- current imbibition phenomena with inclination and dimensionless time and distance. Tables and graphs shows that the saturation of wetting phase be maximum for zero inclination.

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