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Use of Dimensional Analysis to Asses Parameters Correlated to Total Sediment Transport Rate

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Abstract— Sediment transport, and the rate at which it flows, is a complex problem to understand, as the complexity of the nature of the flow is highly dynamic. Due to the reason many researchers have worked for long period of time and have proposed different approaches based on wide range of flume and field data sets and still continue to work. If data sets are used to understand and hypothesis a theory or to generate an equation then predictability and reliability of it depends highly on on accuracy of the data sets. An attempt is made in the present study to find the correlation between various flow parameters and sediment transport rate based on dimensional analysis using B. R. Samaga et al (1986) data set. Total sediment transport rate has been correlated with 13 flow parameters obtained by dimensional analysis which is computed by computer programming performed under Microsoft office, Excell Spread sheet. From the analysis of the selected data set it is observed that the best correlation was obtained between total sediment transport rate and sediment flow parameter $S\,H^2\gamma_s/\omega$

Index Terms— correlation, different approaches, dimensional analysis, flow parameters, predictability, reliability, Sediment transport, sediment transport flow rate.

◆

1 INTRODUCTION

Solutions to model sediment transport, due to their complex nature are determined mostly from experimental and field data predictability and reliability of the developed model depends highly on on accuracy of the data sets. Many reserchers, Yang C. T. (1979), Van Rijn L. C. (1984), Ackers, P. and White, W. R. (1973), Graf, Walter Hans (1971), Engelund, F and Hansen, E (1967), Bagnold, R. A. (1966) etc., have developed total load transport model based on various approaches.

Sediment transport process when studied as a system, large number of operating parameters such as flow and sediment properties parameters is linked as input variables, and a large number of experiments are required to be carried out accordingly to determine the influences of each and every operating parameter on the performance of the system.

S. Q. Yang & S. Y. Lim (2003) developed bed material transport load model based on dimensional analysis considering variours transport parameter and obtained the transport parameter which strongly correlates with the sediment trarnsport rate.S.K. Sinnakaudan et al. (2010) developed total bed material load based on multiple linear regression model and obtained the governing parameters which influences the sediment flow rate.

In order to understand one to one relationship of individual parameter on total sediment load flow rate, data should be generated in a form that one parameter is kept variable and studied that how the flow rate changes with its variation by allowing rest all parameters to be kept as non-varying. Such generation of data is not possible in reality and in natural flow which never happen so neither field experiments nor observed data becomes useful to model sediment transport rate by that approach, also this parameters are interrelated with each other in vivid manner. This pertinent question aroused is answered by the principle of physical similarity and Dimensional analysis.

Dimensional homogeneity of physical quantities implies that the number of dimensionless independent variable is smaller as compared to the number of their dimensionless counterparts to describe the physical phenomena also the interrelationship between individual parameters is sought by this technique because the dimensionless variable represents the criteria of similarity. In the present study, analysis of the transport parameters which correlates strongly with sediment transport rate is done through dimensional analysis using B. R. Samaga et al. (1986).

2 DETAILS OF DATA SET

Belle R. Samaga et al. (1986) conducted experiments in a recirculating tilting flume 30 m (98.3 ft) long, consisting of four mixtures m1, m2, m3 & m4 having different arithmetic mean diameters and geometric standard deviations. The hydraulic data were in the range of 0.06 m ≤ H ≤ 0.11 m, 0.00505 ≤ S ≤ 0.00687, 0.49 m/s ≤ U ≤ 0.73 m/s, arithmetic mean diameter is 0.57 value of Geometric standard deviation is 3.41 for mixture m1. Hydraulic data for m2 mixture range from 0.07 m ≤ H ≤ 0.10 m, 0.00496 ≤ S ≤ 0.00693, 0.53 m/s ≤ U≤ 0.78 m/s, arithmetic mean diameter is 0.55 value of Geometric standard deviation is 3.79. Range of hydraulic data for m 3 mixture is 0.06 m ≤ H ≤ 0.09 m, 0.00542 ≤ S ≤ 0.00693, 0.60 m/s ≤ U ≤ 0.76 m/s; arithmetic mean diameter is 0.42 value of Geometric standard deviation is 2.96. Range of hydraulic data for mixture m 4 is as follows 0.06 m ≤ H ≤ 0.09 m, 0.00449 ≤ S ≤ 0.00610, 0.61 m/s ≤ U ≤ 0.75 m/s; arithmetic mean diameter is 0.25 value of Geometric standard deviation is 1.91.

3 METHODOLOGY

In the present study dimensional analysis is carried out by Buckingum Pi theorem Theorem. *Buckingum Pi theorem states that if a physical problem is described by m dimensional variables, which can be expressed by n fundamental dimensions. Then the*

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number of independent dimensionless variable is known as Pi terms.

This obtained Pi terms are of same dimension and are in the form such that correlation between them is high, as they all represents same physical dimension. From literature review physical phenomena of sediment transport flow rate, g_T is a function of depth of flow channel, width of flow channel, grain shear velocity, slope of surface of water or energy slope, unit weight of sediment particles, viscosity of water, dimension of sediment that is grain size diameter, gravitational acceleration, kinematic viscosity of fluid, fall velocity of the sediment, incipient condition for sediment motion in the flow.

$$g_T = f\ (H, B, U'_*, S, \gamma_s - \gamma, \gamma_s, d, g, \upsilon, \omega, \gamma). \qquad (1)$$

Fundamental dimensions in sediment transport process is mass, length and time and so n = 3. Thus the value of m is 12, and the value of n is 3 and so number of Pi terms which are obtained are 9.

Number of repeating variables as per Buckingum Pi theorm is considerd as 3, $H, U'_*\ \&\ \gamma_s$.

The units used for different variables plays a significant role in deriving the relation under dimensional analysis. So unit system adopted for the present work is MKS unit system which is presented in table number 1.

TABLE 1
UNITS USED FOR DIMENSIONAL ANYSIS

Sr. no.	Parameter	Unit	Dimension
1.	g_T	$Kg/_{m\ s}$	$M^1 L^{-1} T^{-1}$
2.	H, B, d	m	$M^0 L^1 T^0$
3.	U'_*, ω	$m/_s$	$M^0 L^1 T^{-1}$
4.	S	Unit-Less	$M^0 L^0 T^0$
5.	g	$m/_{s^2}$	$M^0 L^1 T^{-2}$
6.	υ	$m^2/_s$	$M^0 L^2 T^{-1}$
7.	γ_s, γ	$Kg/_{m^2\ s^2}$	$M^1 L^{-2} T^{-2}$

3.1 Implicit expression.

Implicit expression of obtained Pi terms by doing dimensional analysis.

$$f_1\ (\frac{g_T\, U'_*}{H^2\, \gamma_s}, \frac{B}{H}, \frac{U'^2_*\, g}{H}, S, \frac{\gamma_s}{\gamma_s - \gamma}, \frac{\gamma}{\gamma_s}, \frac{U'_*\, H}{\upsilon}, \frac{H}{d}, \frac{U'_*}{\omega}.) = 0 \qquad (2)$$

3.1 Explicit expression.

When this implicit expression is explicitly obtained it can be expressed as.

$$\frac{g_T\, U'_*}{H^2\, \gamma_s} = f_2\ (\frac{B}{H}, \frac{U'^2_*\, g}{H}, S, \frac{\gamma_s}{\gamma_s - \gamma}, \frac{\gamma}{\gamma_s}, \frac{U'_*\, H}{\upsilon}, \frac{H}{d}, \frac{U'_*}{\omega}.) \qquad (3)$$

3.3 Correlation coefficient

Correlation coefficient for $g_T\, U'_*/H^2\, \gamma_s$ and different Pi terms shown under equation 3 is found for different data sets. Few Pi terms showed very strong values of correlation, but when two highly correlating terms are multiplied to form a new Pi term its Correlation Value has been seen even more stronger (nearest to one)

So the new explicit function obtained.

$$\frac{g_T\, U'_*}{H^2\, \gamma_s} = f_3\ (\frac{B}{H}, \frac{U'^2_*\, g}{H}, S, \frac{\gamma_s}{\gamma_s - \gamma}, \frac{\gamma}{\gamma_s}, \frac{U'_*\, H}{\upsilon}, \frac{H}{d}, \frac{U'_*}{\omega}, \frac{S\, U'_*}{\omega}, \frac{U'^2_*\, g\, S}{H}, \frac{U'^3_*\, g}{H\, \omega}, \frac{U'_*\, H\, S}{\upsilon}, \frac{B\, S}{H}) \qquad (4)$$

By making g_T as Subject in above expression g_T can be written as.

$$g_T = f_4\ (\frac{B\, H\, \gamma_s}{U'_*}, \frac{U'_*\, H\, \gamma_s}{g}, \frac{S\, H^2\, \gamma_s}{U'_*}, \frac{H^2\, {\gamma_s}^2}{U'_*\, (\gamma_s - \gamma)}, \frac{\gamma\, H^2}{U'_*}, \frac{H^3\, \gamma_s}{\upsilon}, \frac{H^3\, \gamma_s}{d\, U'_*}, \frac{H^2\, \gamma_s}{\omega}, \frac{S\, H^2 \gamma_s}{\omega}, \frac{U'_*\, H\, S\, \gamma_s}{g}, \frac{{U'_*}^2 H\, \gamma_s}{g\, \omega}, \frac{H^3\, S\, c\gamma_s}{\upsilon}, \frac{B\, H\, S\, \gamma_s}{U'_*}) \qquad (5)$$

When g_T is plotted with each and every term under the Equation 5, linear relationship gives better result. In order to measure the strength of the relationship, computation of correlation coefficient is done, which is a numerical indicator of the strength and direction (positive or negative) of the linear relationship between two variables.

Correlation coefficient is computed using karl Person's method as given in equation 6. For precise and accurate result, square of the Pearson product moment correlation coefficient, r^2 is considered for present study. The formula for it for two variables, X and Y is:

Correlation coefficient,

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\ \sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}. \qquad (6)$$

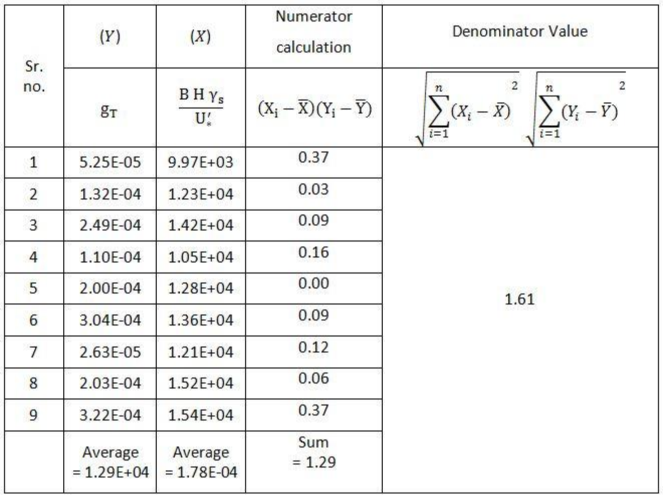
Where:

X and Y are individual observations (e.g. the value of for g_T for ith observation and the value of $B\, H\, \gamma_s/U'_*$ for the same ith observation) and $\bar{X}$ and $\bar{Y}$ are the means for variables X and Y (e.g. the mean value in g_T and $B\, H\, \gamma_s/U'_*$); Sample calculation of r^2 is hown under table 2.

Pearson r coefficient varies between –1 and +1, with +1 indicating a perfect positive relationship (a high score on variable X = a high score on variable Y), -1 a perfect negative relationship (a high score on X = a low score on Y), and 0 indicates as no relationship is present. Thus, in present study most influencing parameter is calculated using equation 5.

TABLE 2

SAMPLE CALCULATION, FOR COMPUTING PEARSON PRODUCT MOMENT CORRELATION COEFFICIENT



Sr. no.	(Y)	(X)	Numerator calculation	Denominator Value
	g_T	$\frac{B\,H\,\gamma_s}{U'_*}$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\ \sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$
1	5.25E-05	9.97E+03	0.37	1.61
2	1.32E-04	1.23E+04	0.03	
3	2.49E-04	1.42E+04	0.09	
4	1.10E-04	1.05E+04	0.16	
5	2.00E-04	1.28E+04	0.00	
6	3.04E-04	1.36E+04	0.09	
7	2.63E-05	1.21E+04	0.12	
8	2.03E-04	1.52E+04	0.06	
9	3.22E-04	1.54E+04	0.37	
	Average = 1.29E+04	Average = 1.78E-04	Sum = 1.29	

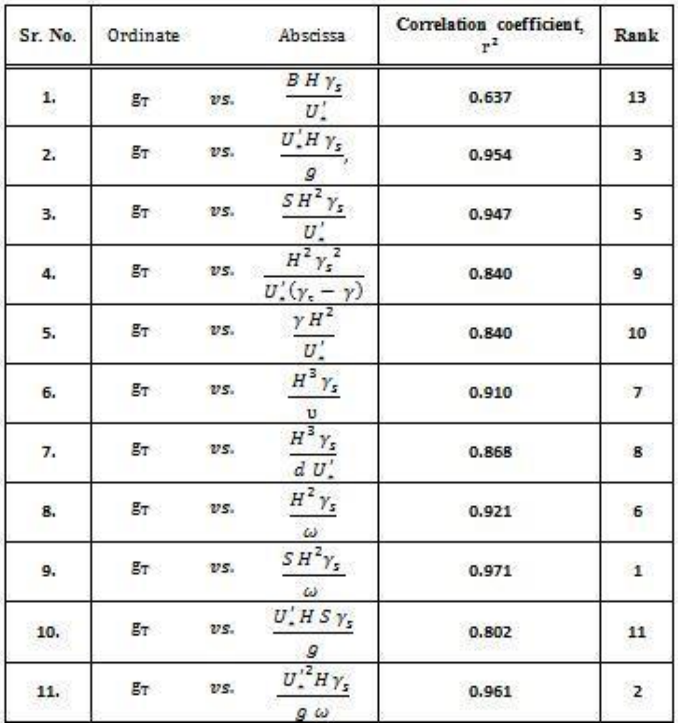
r = 1.29/1.61, that gives a value of r as 0.801 & r^2 as 0.64

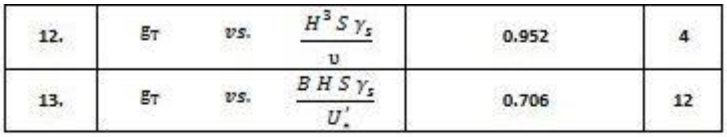
4 RESULT AND ANALYSIS

Correlation coefficient of g_T with all thirteen Pi terms of equation 5 for Belle R. Samaga et al. mixtue m1, m2, m3 & m4 are presented in table 3, table 4, table 5 & table 6 respectively. The value of correlation coefficient found most near to 1 among these thirteen values, has been given the rank 1 and the one next to it as rank 2 and so on till rank 13 which showed value which is closest to zero and have least correlation.

TABLE 3

COMPARING CORRELATION COEFFICIENT VALUES, DATA SET NAME: BELLE R. SAMAGA ET AL. MIXTURE - 1



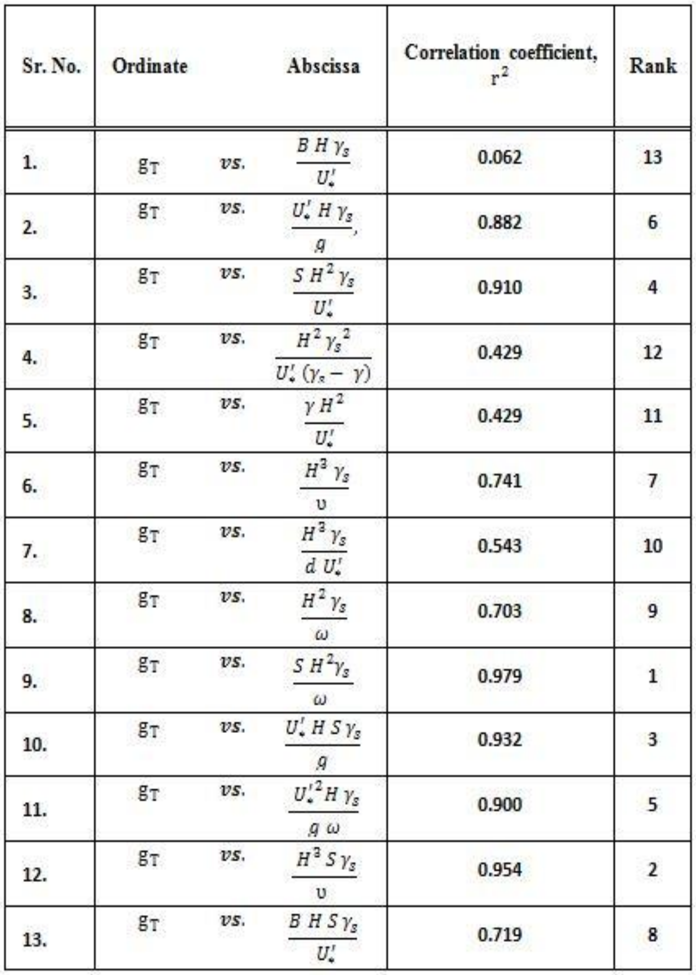


Sr. No.	Ordinate		Abscissa	Correlation coefficient, r^2	Rank
1.	g_T	vs.	$\frac{B\,H\,\gamma_s}{U'_*}$	0.637	13
2.	g_T	vs.	$\frac{U'_* H\,\gamma_s}{g}$,	0.954	3
3.	g_T	vs.	$\frac{S\,H^2\,\gamma_s}{U'_*}$	0.947	5
4.	g_T	vs.	$\frac{H^2\,\gamma_s{}^2}{U'_*(\gamma_s - \gamma)}$	0.840	9
5.	g_T	vs.	$\frac{\gamma\,H^2}{U'_*}$	0.840	10
6.	g_T	vs.	$\frac{H^3\,\gamma_s}{\upsilon}$	0.910	7
7.	g_T	vs.	$\frac{H^3\,\gamma_s}{d\,U'_*}$	0.868	8
8.	g_T	vs.	$\frac{H^2\,\gamma_s}{\omega}$	0.921	6
9.	g_T	vs.	$\frac{S\,H^2\gamma_s}{\omega}$	0.971	1
10.	g_T	vs.	$\frac{U'_* H\,S\,\gamma_s}{g}$	0.802	11
11.	g_T	vs.	$\frac{U'^2_* H\,\gamma_s}{g\,\omega}$	0.961	2
12.	g_T	vs.	$\frac{H^3\,S\,\gamma_s}{\upsilon}$	0.952	4
13.	g_T	vs.	$\frac{B\,H\,S\,\gamma_s}{U'_*}$	0.706	12

By Comparing correlation coefficient values, for data set B. R. Samaga et al. Mixture m1, $S\,H^2\gamma_s/\omega$ ranks highest with the value of 0.971 followed by $U'^2_* H\,\gamma_s/g\,\omega$ having value 0.961, $U'_*\,H\,\gamma_s/g$ having value of 0.954 etc.

TABLE 4

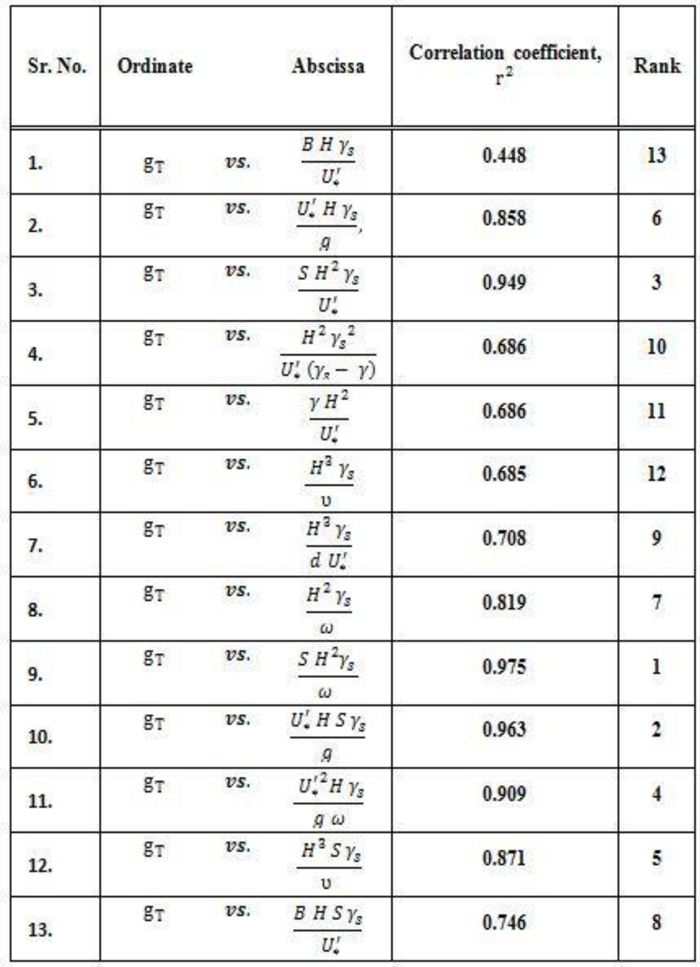
COMPARING CORRELATION COEFFICIENT VALUES, DATA SET NAME: BELLE R. SAMAGA ET AL. MIXTURE - 2



Sr. No.	Ordinate		Abscissa	Correlation coefficient, r^2	Rank
1.	g_T	vs.	$\frac{B\,H\,\gamma_s}{U'_*}$	0.062	13
2.	g_T	vs.	$\frac{U'_*\,H\,\gamma_s}{g}$,	0.882	6
3.	g_T	vs.	$\frac{S\,H^2\,\gamma_s}{U'_*}$	0.910	4
4.	g_T	vs.	$\frac{H^2\,\gamma_s{}^2}{U'_*\,(\gamma_s - \gamma)}$	0.429	12
5.	g_T	vs.	$\frac{\gamma\,H^2}{U'_*}$	0.429	11
6.	g_T	vs.	$\frac{H^3\,\gamma_s}{\upsilon}$	0.741	7
7.	g_T	vs.	$\frac{H^3\,\gamma_s}{d\,U'_*}$	0.543	10
8.	g_T	vs.	$\frac{H^2\,\gamma_s}{\omega}$	0.703	9
9.	g_T	vs.	$\frac{S\,H^2\gamma_s}{\omega}$	0.979	1
10.	g_T	vs.	$\frac{U'_*\,H\,S\,\gamma_s}{g}$	0.932	3
11.	g_T	vs.	$\frac{U'^2_*\,H\,\gamma_s}{g\,\omega}$	0.900	5
12.	g_T	vs.	$\frac{H^3\,S\,\gamma_s}{\upsilon}$	0.954	2
13.	g_T	vs.	$\frac{B\,H\,S\,\gamma_s}{U'_*}$	0.719	8

For Mixture m2, $S\,H^2\gamma_s/\omega$ rank highest with the value of 0.797 followed by $H^3\,S\,\gamma_s/\upsilon$ 0.954, $U'_*\,H\,\gamma_s/g$ having value of 0.932 etc

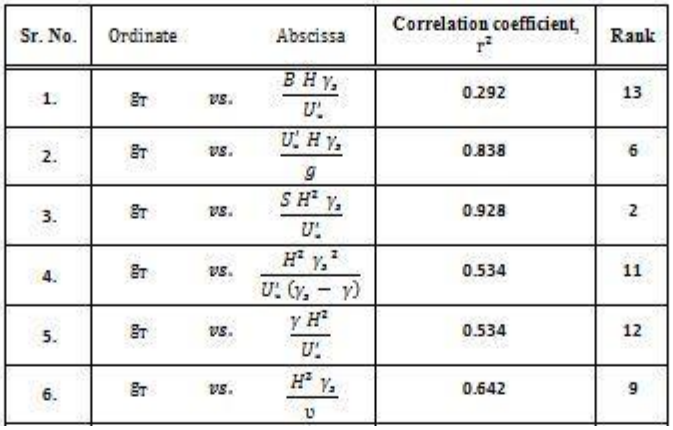
TABLE 5
COMPARING CORRELATION COEFFICIENT VALUES, DATA SET NAME: BELLE R. SAMAGA ET AL. MIXTURE - 3

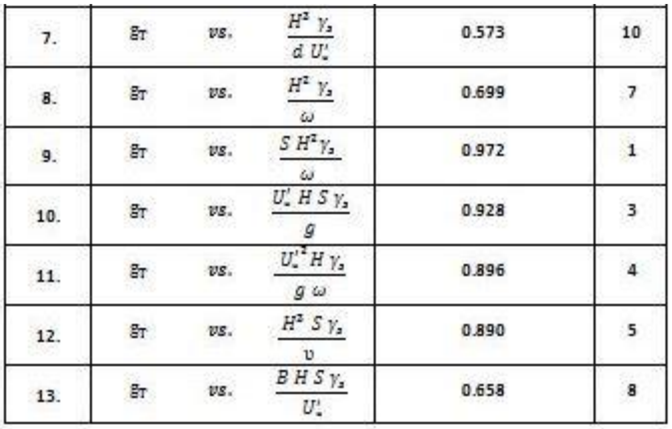


Sr. No.	Ordinate		Abscissa	Correlation coefficient, r^2	Rank
1.	g_T	vs.	$\frac{B\,H\,\gamma_s}{U'_*}$	0.448	13
2.	g_T	vs.	$\frac{U'_*\,H\,\gamma_s}{g}$,	0.858	6
3.	g_T	vs.	$\frac{S\,H^2\,\gamma_s}{U'_*}$	0.949	3
4.	g_T	vs.	$\frac{H^2\,{\gamma_s}^2}{U'_*\,(\gamma_s-\gamma)}$	0.686	10
5.	g_T	vs.	$\frac{\gamma\,H^2}{U'_*}$	0.686	11
6.	g_T	vs.	$\frac{H^3\,\gamma_s}{\upsilon}$	0.685	12
7.	g_T	vs.	$\frac{H^3\,\gamma_s}{d\,U'_*}$	0.708	9
8.	g_T	vs.	$\frac{H^2\,\gamma_s}{\omega}$	0.819	7
9.	g_T	vs.	$\frac{S\,H^2\gamma_s}{\omega}$	0.975	1
10.	g_T	vs.	$\frac{U'_*\,H\,S\,\gamma_s}{g}$	0.963	2
11.	g_T	vs.	$\frac{{U'_*}^2\,H\,\gamma_s}{g\,\omega}$	0.909	4
12.	g_T	vs.	$\frac{H^3\,S\,\gamma_s}{\upsilon}$	0.871	5
13.	g_T	vs.	$\frac{B\,H\,S\,\gamma_s}{U'_*}$	0.746	8

For Mixture m3, $S\,H^2\gamma_s/\omega$ ranks highest with the value of 0.975 followed by $U'_*\,H\,\gamma_s/g$ with the value of 0.963, $S\,H^2\gamma_s/U'_*$ having value of 0.949.

TABLE 6
COMPARING CORRELATION COEFFICIENT VALUES, DATA SET NAME: BELLE R. SAMAGA ET AL. MIXTURE - 4





Sr. No.	Ordinate		Abscissa	Correlation coefficient, r^2	Rank
1.	g_T	vs.	$\frac{B\,H\,\gamma_s}{U'_*}$	0.292	13
2.	g_T	vs.	$\frac{U'_*\,H\,\gamma_s}{g}$	0.838	6
3.	g_T	vs.	$\frac{S\,H^2\,\gamma_s}{U'_*}$	0.928	2
4.	g_T	vs.	$\frac{H^2\,{\gamma_s}^2}{U'_*\,(\gamma_s-\gamma)}$	0.534	11
5.	g_T	vs.	$\frac{\gamma\,H^2}{U'_*}$	0.534	12
6.	g_T	vs.	$\frac{H^3\,\gamma_s}{\upsilon}$	0.642	9
7.	g_T	vs.	$\frac{H^3\,\gamma_s}{d\,U'_*}$	0.573	10
8.	g_T	vs.	$\frac{H^2\,\gamma_s}{\omega}$	0.699	7
9.	g_T	vs.	$\frac{S\,H^2\gamma_s}{\omega}$	0.972	1
10.	g_T	vs.	$\frac{U'_*\,H\,S\,\gamma_s}{g}$	0.928	3
11.	g_T	vs.	$\frac{{U'_*}^2\,H\,\gamma_s}{g\,\omega}$	0.896	4
12.	g_T	vs.	$\frac{H^3\,S\,\gamma_s}{\upsilon}$	0.890	5
13.	g_T	vs.	$\frac{B\,H\,S\,\gamma_s}{U'_*}$	0.658	8

For Mixture m4, $S\,H^2\gamma_s/\omega$ ranks highest with the value of 0.972 followed by $S\,H^2\gamma_s/U'_*$ with the value of 0.928, $U'_*\,H\,\gamma_s/g$ having value of 0.928.

5 CONCLUSION

The most influencing parameter for Total sediment flow rate, g_T. (Kg/m s) out of 13 different parameters under equation 5 is $S\,H^2\gamma_s/\omega$ (Kg/m s) with correlation coefficient's value of 0.971, 0.979, 0.975 & 0.972 for Belle R. Samaga et al. data set's mixture m1 , m2, m3 & m4.

Notation

B = width of channel;
d = sediment diameter;
g_T = total sediment flow rate (kg/ms);
g = gravitational acceleration;
H = water depth;
Q = flow discharge;
r = Pearson product moment correlation coefficient;
S = energy slope;
U = Average velocity.
u_* = shear velocity
U'_* = shear velocity due to grain;
U_{*c} = Shields critical shear velocity;
γ = specific weight of water;
γ_s = specific weight of sediment;
υ = kinematic viscosity of water;
ω = fall velocity of particle.

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