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MATHEMATICAL MODELING FOR VALIDATION OF EXPERIMENTAL WORK

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Abstract - *A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling. Mathematical models are used in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines (such as computer science, artificial intelligence) as well as in the social sciences (such as economics ,psychology, sociology, etc.) Physicists, engineers, statisticians operation research analysts, and economists use mathematical models most extensively. A model may help to explain a system and to study the effects of different components, and to make predictions about behavior.*

I. INTRODUCTION

A mathematical model is an abstract model that uses mathematical language to describe the behavior of a system. Mathematical model is a representation in mathematical terms of the behavior of real devices and objects Mathematical models are used particularly in the natural sciences and engineering disciplines (such as physics, biology, and electrical engineering) but also in the social sciences (such as economics, sociology and political science); physicists, engineers, computer scientists, and economists use mathematical models most extensively. Eykhoff (1974) defined a mathematical model as 'a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable for'. Mathematical models can take many forms, including but not limited to dynamical systems, statistical models, differential equations, or game theoretic models. These and other types of models can overlap, with a given model involving a variety of abstract structures. Mathematical modeling problems are often classified into black box or white box models, according to how much a priori information is available of the system. A black-box model is a system of which there is no a priori information available. A white-box model (also called glass box or clear box) is a system where all necessary information is available.

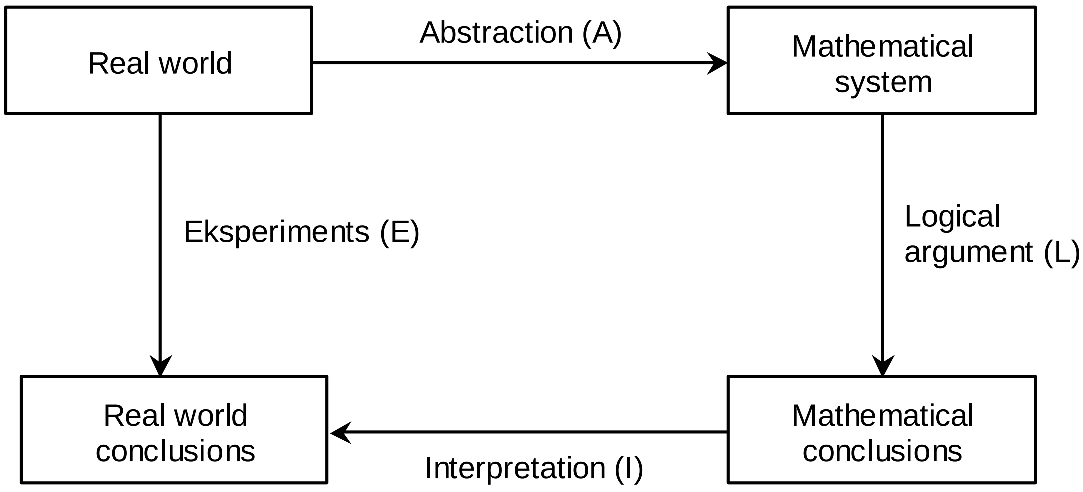
II. NEED OF MATHEMATICAL MODELING

A mathematical model embodies a hypothesis about the study system, and lets you compare that hypothesis with data. A model is often most useful when it fails to fit the data, because that says that some of your ideas about the study system are wrong. Mathematical models and computer simulations are useful experimental tools for building and testing theories, assessing quantitative conjectures, answering specific questions, determining sensitivities to changes in parameter values and estimating key parameters from data. A model is a representation or an abstraction of a system or a process. We build models because they help us to (1) define our problems, (2) organize our thoughts, (3) understand our data, (4) communicate and test that understanding, and (5) make predictions. A model is therefore an intellectual tool.

One of the most important aims for construction of models is to define the problem such that only important details becomes visible, while irrelevant features are neglected. A road map of the triangle area is an example of a model. If a motorist understands the symbols that are used in the map, then much information about the region becomes available in a package small enough to carry around in one's pocket.. The road map is one representation of many important features of the region. But it omits many other features that may not be crucial. Most road maps do not contain sufficient information to tell a motorist what is the speediest route to take between two points during the morning rush hour. The map is also almost useless to a door-to-door encyclopedia salesperson who wishes to find neighborhoods whose social and economic characteristics indicate good selling opportunities. For this purpose, a different kind of model of the region is needed.

III. MATHEMATICAL MODELING BACKGROUND

A mathematical model of a complex phenomenon or situation has many of the advantages and limitations of other types of models. Some factors in the situation will be omitted while others are stressed. When constructing a mathematical system, the modeler must keep in mind the type of information he or she wishes to obtain from it. The role that mathematical models play in science can be illustrated by the relatively simple schematic diagram of Figure 1.



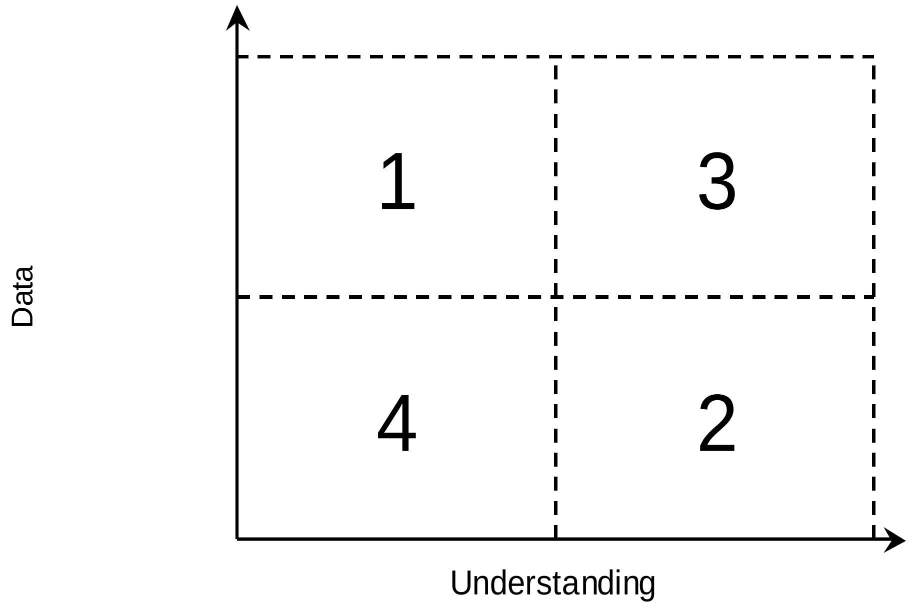
The scientist begins with some observations about the real world. He or she wishes to make some conclusions or predictions about the situation he or she has observed. One way to proceed (E) is to conduct some experiments and record the results. The model builder follows a different path. First, he or she abstracts, or translates, some of the essential features of the real world into a mathematical system. Then by logical argument (L) he or she derives some mathematical conclusions. These conclusions are then interpreted (I) as predictions about the real world. To be useful, the mathematical system should predict conclusions about the real world that are actually observed when appropriate experiments are carried out. If the predictions from the model bear little resemblance to what actually occurs in the real world, then the model is not a good one. The modeler has not isolated the critical features of the situation being studied or the axioms misrepresent the relations among these features. On the other hand, if there is good agreement between what is observed and what the model predicts, then there is some reason to believe that the mathematical system does indeed capture correctly important aspects of the real-world situation.

What happens quite frequently is that some of the predictions of a mathematical model agree quite closely with observed events, while other predictions do not agree with the observed events. In such a case, we might hope to modify the model to improve its accuracy. The incorrect predictions may suggest ways of rethinking the assumptions of the mathematical system. One hopes that the revised model will not only preserve the correct predictions of the original one, but that it will also make further correct predictions. The incorrect inferences of the revised model will lead, in turn, to yet another version, more sophisticated more accurate than the previous one.

However, it is important to keep in mind, that the goal is not to make the most precise model of the part of the world that is modeled, but that the model (like the road map) includes all the essential features, even if that means that some other features in the model do not present the reality. For example, a model of the cardiovascular system (the heart, arteries, and veins) could accurately present the systemic arteries and veins and then lump the pulmonary circulation into a single compartment. Such a compartment would never represent any of the subsystems correctly.

When building mathematical models one should distinguish between the different types of models, some models (deterministic models) can be derived directly from physical laws (e.g. Newton's second law), while other models are based on empirical observations. Both types of models provide insight into the system modeled, but the type of model must be considered carefully. For example, very different types of models are used for predict the weather tomorrow and to determine a rockets trajectory to the moon.

Holling (1978) has a diagram (Figure 2) that provides a simple and useful classification of problems. The horizontal axis represents how well we understand the problem we are trying to solve; the vertical axis represents the quality and/or quantity of relevant data (Figure 2). Holling divides the quadrant between the two axes into four areas, corresponding to four classes of problems.



Area 1 is a region with good data but little understanding. This is where statistical techniques are useful; they enable one to analyze the data search for patterns or relation, construct and test hypotheses, and so on.

Area 3 is a region with good data and good understanding. Many problems in engineering and the physical sciences (for example, the problem of computing a rockets trajectory to the moon) belong to this class of problems. This is the area where models are used routinely and with confidence because their effectiveness has been proved repeatedly.

Area 2 has little in the way of supporting data but there is some understanding of the structure of the problem.

Area 4, in this area there is little knowledge of the structure of the problem and little data to support it.

Unfortunately, many problems in the nonphysical sciences (especially in the biological sciences) belong to areas 2 and 4. However, recent explosion in experimental techniques move some of these problems to areas 1 and 3. The main difference from the physical problems is the uncertainty and high levels of noise often found in the data.

The modeling challenges for problems in area 2 and 4 are:

- Decisions may have to be made despite the lack of data and understanding. How do we make good, scientific decision under these circumstances
- How do we go about improving our understanding and suggest new ways for collecting the data necessary to validate the modeling. This is an area where modeling can be used to predict new experimental settings.

Models that lie in areas 2 and 4 are bound to be speculative. They will never have the respectability of models build for solving problems in area 3 because it is unlikely they will be sufficiently accurate of that they can ever be tested conclusively. In fact most models in biology cannot be tested conclusively, while we have a lot more data today that 20 years ago there are still many types of data that could validate models, but that are unethical to measure. Models build this way should never be used unquestioningly or automatically. The whole process of building and using these models has to be that much more thoughtful because we do not really understand the structure of the problem and do not have (and cannot easily get) supporting data.

We therefore build models to explore the consequences of what we believe to be true. Those who have a lot of data and little understanding of their problem (area 1) gain understanding by “living with” their data, looking at it in different ways, and searching for patterns and relationships. Because we have so little data in areas 2 and 4, we learn by living with our models, by exercising them, manipulating them, questioning their relevance, and comparing their behavior with what we know (or think we know) about the real world. This process often forces us to reevaluate our beliefs, and that reevaluation in turn leads to new versions off the models. The mere act of assembling the pieces and building a model (however speculative the model might be) usually improves our understanding and enables us to find or use data we had not realized were relevant. That in turn leads us to a better model. The process is one of boot-strapping: If we begin with little data and understanding in the bottom left-hand corner of Holling's diagram, models help us to zigzag upwards and to the right. This is a far healthier approach than one of just collecting data because we improve our understanding as we go along. (Those who collect data without building models run the very real risk of discovering, when they eventually analyze their data, that they have collected the wrong data!)

IV. THE MODELING PROCESS:

Before starting the mathematical modeling we have to go through following principal questions and answers.

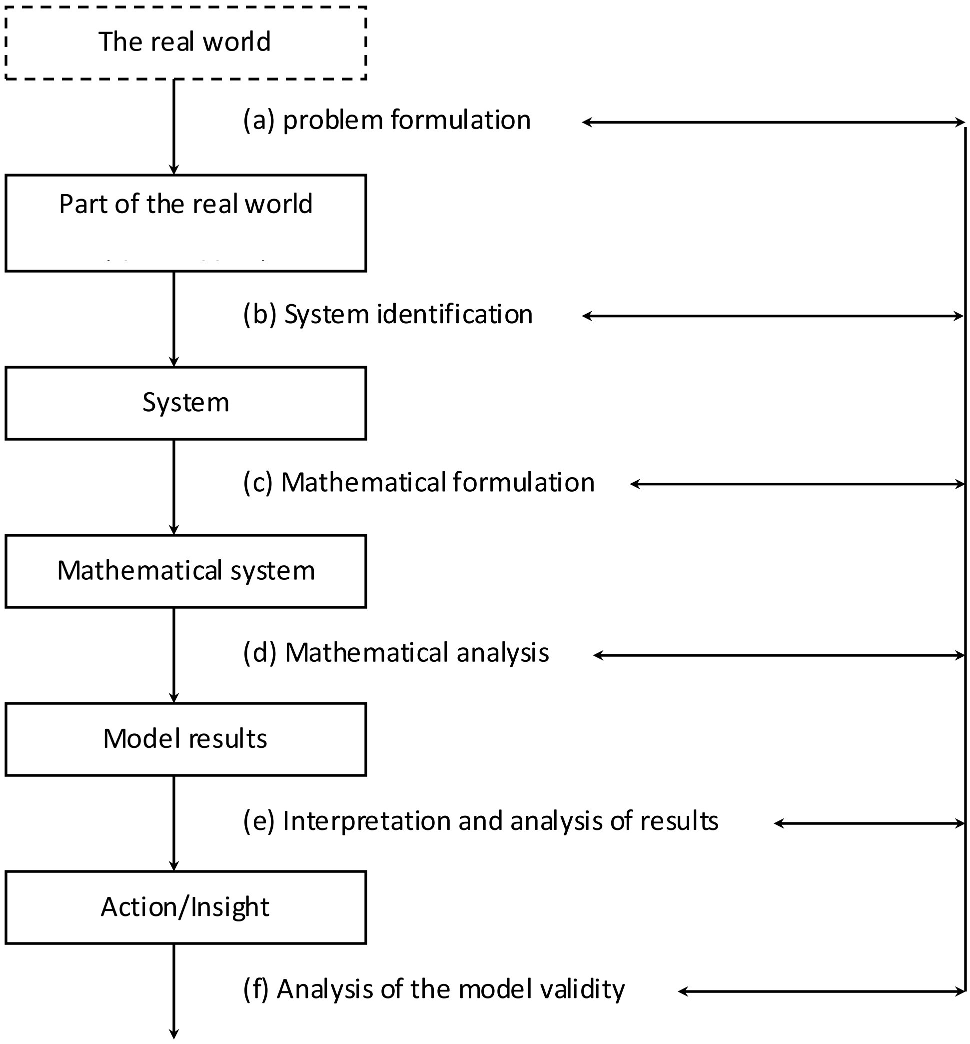
• **Why? -** What are we looking for? Identify the need for the model.
• **Find? -** What do we want to know? List the data we are seeking.
• **Given? -** What do we know? Identify the available relevant data.
• **Assume? -**What can we assume? Identify the circumstances that apply.
• **How? -** How should we look at this model? Identify the governing Physical principles.
• **Predict?** What will our model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
• **Valid?** Are the predictions valid? Identify tests that can be made to *validate* the model, i.e., is it consistent with its principles and assumptions?
• **Verified?** Are the predictions good? Identify tests that can be made to *verify* the model, i.e., is it useful in terms of the initial reason it was done?
• **Improve?** Can we improve the model? Identify parameter values that are not adequately known, variables that should have been included, and/or assumptions/restrictions that could be lifted. Implement the iterative loop thatwe can call "model-validate-verify-improve-predict."
• **Use?** How will we exercise the model? What will we do with the model?
After Summarizing all the existing ideas and in order to make the ideal model a bit clearer , we could say that the main stages of the mathematical modeling process involve:
• s1 = analysis of the problem (understanding the statement and recognizing the restrictions and requirements of the real system).

• s2 = mathematization, which could be divided to formulation of the real situation in such a way that it will be ready for mathematical treatment and construction of the model. The former involves a deep abstracting process, in order to transfer from the real system to the, so called, assumed real system, where emphasis is given to certain, dominating for the system's performance, variables.

• s3 = solution of the model, which is achieved by proper mathematical manipulation.

• s4 = validation (control) of the model, which is usually achieved by reproducing, through the model, the behavior of the real system under the conditions existing before the solution of the model (empirical results, special cases etc). A model is valid, if despite its inexactness in representing the real system, gives a reliable prediction of the system's performance.

• s5 = Implementation of the final mathematical results to the real system.



V. SOME METHODS OF MATHEMATICAL MODELING

5.1. Dimensional Homogeneity and Consistency

There is a basic, yet very powerful idea that is central to mathematical modeling, namely, that every equation we use must be dimensionally homogeneous or dimensionally consistent. It is quite logical that every term in an energy equation has total dimensions of energy, and that every term in a balance of mass should have the dimensions of mass. This statement provides the basis for a technique called dimensional analysis

5.2 Abstraction and Scaling

An important decision in modeling is choosing an appropriate level of detail for the problem at hand, and thus knowing what level of detail is prescribed for the attendant model. This process is called *abstraction* and it typically requires a thoughtful approach to identifying those phenomena on which we want to focus, that is, to answering the fundamental question about why a model is being sought or developed.For example, a linear elastic spring can be used to model more than just the relation between force and relative extension of a simple coiled spring, as in an old-fashioned butcher's scale or an automobile spring. It can also be used tomodel the static and dynamic behavior of a tall building, perhaps to model wind loading, perhaps as part of analyzing how the building would respond to an earthquake. In these examples, we can use a very abstract model by subsuming various details within the parameters of that model.

5.3 Conservation and Balance Principles

When we develop mathematical models, we often start with statements that indicate that some property of an object or system is being conserved. For example, we could analyze the motion of a body moving on an ideal, frictionless path by noting that its energy is *conserved*. Sometimes, as when we model the population of an animal colony or the volume of a river flow, we must *balance* quantities, of individual animals or water volumes, that cross a defined boundary. We will apply *balance* or *conservation principles* to assess the effect of maintaining or conserving levels of important physical properties. Conservation and balance equations are related—in fact, conservation laws are special cases of balance laws.

5.4 Constructing Linear Models

Linearity is one of the most important concepts in mathematical modeling. Models of devices or systems are said to be *linear* when their basic equations—whether algebraic, differential, or integral—are such that the magnitude of their behavior or response produced is *directly proportional* to the excitation or input that drives them. Even when devices like the pendulum are more fully described by nonlinear models, their behavior can often be approximated by linearized or perturbed models, in which cases the mathematics of linear systems can be successfully applied. We apply linearity when we model the behavior of a device or system that is forced or pushed by a complex set of inputs or excitations. We obtain the response of that device or system to the sum of the individual inputs by adding or *superposing* the separate responses of the system to each individual input. This important result is called the *principle of superposition*. Engineers use this principle to predict the response of a system to a complicated input by decomposing or breaking down that input into a set of simpler inputs that produce known system responses or behaviors.

VI. CONCLUSION

The mathematical modeling of devices and phenomena is essential in both engineering and science; engineers and scientists have very practical reasons for doing mathematical modeling. In addition, engineers, scientists, and mathematicians want to experience the sheer joy of formulating and solving mathematical problems. It is most important to remember that mathematical models are representations or descriptions of reality—by their very nature they *depict reality* and hence We want to know how to make or generate mathematical representations or models, how to validate them, how to use them, and how and when their use is limited.

REFERENCES

[1]. "An Introduction to Mathematical Models in the Social and Life Sciences", Michael Olinick, Addison-Wesley Publishing Company, Massachusetts, 1978

[2]. "BASE Note 1", Morten Blomhoj, Tinne Hoff Kjeldsen, Johnny Ottesen, Natural Sciences Basis Program, Roskilde University Center, Denmark, August 2000

[3]. "Building Models for Conservation and Wildlife Management", Second edition, A.M. Starfield, A.L. Bleloch, Burgess International Group Inc., Minnesota, 1991

[4]. J.R.Thomson Engineering Safety Assessment. Longman Scientific and Technical Hong Kong, 1987.

[5]. Lee F.P., Loss Prevention in the Process Industries, Butterworths, 1980.

[6]. National Mathematical Centre (NMC): On establishment of Ecological Research Unit at NMC for ecological forecasting and solution to Nigerian ecological problems.Proj.Pro.No.2C, January 2003

[7]. *Modelling Methodology for Physiology andMedicine* E. Carson and C. Cobelli (Eds, Academic Press, San Diego, CA, 2002

[8]. *Fundamentals of Modeling and Analyzing Engineering Systems*.P. D. Cha, J. J. Rosenberg, and C. L. Dym, , Cambridge University Press, New York, 2000.

[9]. , *Engineering Design: A Synthesis of Views* C. L. Dym, Cambridge University Press, New York, 1994.

[10]. , *Principles of Mathematical Modeling* C. L. Dym and E. S. Ivey, *Principles of Mathematical Modeling*, 1st Edition, Academic Press, New York, 1980

[11]. *Language in Thought and Action* S. I. Hayakawa, *Language in Thought and Action*, Harcourt, Brace, New York, 1949.

