

**COMBINED ECONOMIC EMISSIONS DISPATCH PROBLEM USING
LAGRANGE'S METHOD**Hemanshu C. Mahla¹, Prof. R. H. Bhesdadiya², Pradeep Jangir³, Asso. Prof. I. N. Trivedi⁴¹ PG Student, Electrical Department, LE College, Gujarat, India² Professor, Electrical Department, LE College, Gujarat, India³ PG Student, Electrical Department, LE College, Gujarat, India⁴ Professor, Electrical Department, LE College, Gujarat, India

Abstract -In this paper Lagrange's algorithm is proposed to optimize Economic Emission Dispatch Problem (EED) of thermal units. Economic Emission Dispatch Problem is used to minimize both total fuel cost of generation and the emission of toxic gases of the thermal generating units. The bi-objective problem is converted into single objective problem by introducing price penalty factor to maintain an acceptable system performance in terms of limits on generator real power outputs, with minimum emission dispatch. In this paper, the proposed algorithm has been applied to a standard IEEE 30 bus test system with six generating units. The total fuel cost of the economic emission dispatch problem is less by using Min-Max price penalty factor approach in comparison to Max-Max price penalty factor.

Keywords: Lagrange's method, Combined economic emission dispatch, fuel cost function, emission function, price penalty factor

I. INTRODUCTION

In electrical power system the main aim is to schedule the better generating units to meet the required load demand without violating constraints. But the thermal power plant play major role in power production and they burn fossil fuel that generating toxic gases. This toxic gases creates pollution in the environment. The minimum cost condition corresponds to minimum cost with considerable amount of emission. Similarly the emission minimum condition produces minimum emission with bigger deviation from the minimum cost. These two condition cannot be consider similarly. The feasible optimum corresponds to a small deviation in cost with an allowable tolerance in emission taking into account emission constraints. This termed is known as emission constrained economic dispatch [1-11]. It is compulsory for the electric utilities to reduce the pollution level by reducing production of Sulphur dioxide, nitrogen oxide and carbon dioxide gases. The paper solves an economic emissions dispatch problem using Lagrange's method by similarly minimizing the operational costs and pollutant emissions.

Many research paper used solving the economic emissions dispatch problem using various price penalty factor. But most of the paper [6-12] used the max-max price penalty factor to solve the combined economic emissions dispatch problem.

The optimization problem is calculated as price penalty factor h_i [6]. The price penalty factor h_i is the ratio of the maximum fuel cost and maximum emissions values of corresponding generator [10] and it is expressed as

$$h_i = \frac{F_c(P_{i,max})}{E_c(P_{i,min})}, \quad i=1, n(1)$$

Where $F_c(P_{i,max})$ is the maximum fuel cost, $E_c(P_{i,min})$ is the maximum emission, n is the number of generating units i , P_i is the active power produced by the i^{th} generator

The above definition gives an appropriate value of price penalty factor for the corresponding load demand. The sequential approach with matrix framework method is used in [12] for single area power dispatch problem using Max-Max price penalty factor. The Max-Max price penalty factor converts the multi-objective optimization problem into a single objective optimization problem. For the case of quadratic type cost functions the maximum price penalty factor of each generator is the ratio between the fuel cost and emission at its maximum power output as [12]

$$h_i = \frac{a_i P_{i,max}^2 + b_i P_{i,max} + c_i}{d_i P_{i,max}^2 + e_i P_{i,max} + f_i} [\$/kg] (2)$$

Where

a_i, b_i, c_i are the cost coefficients, d_i, e_i, f_i are the emission coefficients, $P_{i,max}$ is the maximum real power output of the generating unit i

II. ECONOMIC DISPATCH PROBLEM

The main objective of economic dispatch is to minimize the generating cost rate and to meet the required load demand without violating. The main objective of the economic emissions dispatch problem formulated as

$$\text{Minimize } F_C = \sum_{i=1}^n F_i(P_i) = \sum_{i=1}^n (a_i P_{i,max}^2 + b_i P_{i,max} + c_i) \text{ [$/hr]} \quad (3)$$

Where

F_C - Total Fuel Cost, $F_i(P_i)$ - Fuel cost of the i^{th} generator, P_i - Real power generation of unit i

a_i, b_i, c_i - Cost coefficients of generating for unit i , n - Number of generating units

Under the constraints

1. Power balance constraint

$$\sum_{i=1}^n P_i = P_G = P_D \quad \text{[MW]} \quad (4)$$

Where

P_G - Total power generation of the system, P_D - Total demand of the system

2. Generator operational constraints

$$P_{i,min} \leq P_i \leq P_{i,max}, \quad i=1, n \quad \text{[MW]} \quad (5)$$

Where

$P_{i,min}$ - Minimum value of real power allowed at generator i , $P_{i,max}$ - Maximum value of real power allowed at Generator i

III. SINGLE AREA DISPATCH PROBLEM

In the thermal power plant the various pollution like sulphur dioxide, nitrogen oxide and carbon dioxide are the major pollutants. The problem is formulated by including the reduction of these emissions as an objective given by the following equation.

$$F_T = \sum_{i=1}^n (d_i P_{i,max}^2 + e_i P_{i,max} + f_i) \text{ [Kg /hr]} \quad (6)$$

Where

F_T - Total emission, d_i, e_i, f_i - Emission coefficients of generating unit i

A multi-objective optimization is converted into a single objective optimization problem by introducing price penalty factor h_i as follows,

$$F_T = \sum_{i=1}^n (a_i P_i^2 + b_i P_i + c_i) + h_i (d_i P_i^2 + e_i P_i + f_i) \text{ [$/hr]} \quad (7)$$

Where

F_T - EED's fuel cost, h_i - Price penalty factor

IV. FORMULATION OF MIN-MAX AND MAX-MAX PRICE PENALTY FACTOR'S FOR CEED PROBLEM

This paper used the min-max and max-max price penalty factor for EED dispatch problem. The ratio of the minimum fuel cost and maximum emissions factor. It is called min-max price penalty factor and is described as:

$$h_i = \frac{a_i P_{i,min}^2 + b_i P_{i,min} + c_i}{d_i P_{i,max}^2 + e_i P_{i,max} + f_i} \text{ [$/kg]} \quad (8)$$

The second max-max price penalty factor is describe as the ratio of the maximum fuel cost and maximum emissions values of the generator units. This price penalty factor is called max-max price penalty factor and is described as:

$$h_i = \frac{a_i P_{i,max}^2 + b_i P_{i,max} + c_i}{d_i P_{i,max}^2 + e_i P_{i,max} + f_i} \text{ [$/kg]} \quad (9)$$

V. LAGRANGE'S METHOD FOR ECONOMIC EMISSION DISPATCH PROBLEM

The method of Lagrange's multipliers (named after Joseph Louis Lagrange) provides a strategy for finding the local maxima or minima of a function subject to the equality or inequality constraints. The problem is solved by introduction of a function of Lagrange based on a Lagrange's multiplier λ , as follows:

$$L = \sum_{i=1}^N [(c_i + b_i P_i + a_i P_i^2) + h_i (f_i + e_i P_i + d_i P_i^2)] + \lambda (P_D - \sum_{i=1}^n P_i) \quad (10)$$

where

a_i, b_i, c_i - cost coefficient, d_i, e_i, f_i - emissions coefficient, λ - Lagrange's multipliers,

P_D - power demand, P_i - real power generation

Necessary conditions for solution of the problem According to P_i ($i=1$ to n) and λ is :

According to P_i $\partial L / \partial P_i = 0$, According to λ $\partial L / \partial \lambda = 0$

At the better solution this gradient has to be equal to zero. As analytical solution is not possible the gradient procedure for calculation of λ has to be developed as follows

$$\lambda^{(k+1)} = \lambda^k + \alpha \Delta \lambda^{(k)}, \lambda \neq 0$$

Where

k is the iteration number and , $k = 0 : m$, m is the given maximum number of iterations.

$\Delta \lambda^{(k)}$ is determined by the equation (10) and α is the step of the gradient procedure. The calculations will start from initial value of the Lagrange's variable $\lambda^{(0)}$. When during the iterations $\Delta \lambda = 0$, the Lagrange's variable is better solution given. It will determine the optimal solution for the power that has to be produced by the generators.

The obtained solutions for $P_i, i = 1, n$ have to belong to the constraint domain (6). That is why for every step of the gradient procedure, the obtained solution is fit to the constraint domain following the procedure

$$P_i^{(k)} = \left\{ \begin{array}{l} P_{i,min}, \text{ if } P_i^{(k)} < P_{i,min}, \\ P_i^{(k)}, \text{ if } P_{i,min} \leq P_i^{(k)} < P_{i,max}, \\ P_{i,max}, \text{ if } P_i^{(k)} > P_{i,max}, \end{array} \right\}$$

The condition for end of the iteration is $\Delta \lambda \leq \varepsilon$

Where $\varepsilon > 0$ is a small number.

The algorithm of the method is:

- 1) Initial value of the Lagrange's multiplier is guessed: $\lambda^{(0)}$, and the value of the condition for optimality ε is given.
- 2) In equation Matrices $E^{(0)}$, and $D^{(0)}$, are formed
- 3) Equation is solved and $P^{(0)} = E^{(0)} \setminus D^{(0)}$ is determined.
- 4) The obtained vector P is fit to the constraints.
- 5) $\Delta \lambda^{(0)}$ is calculated using equation where $P^{(0)}$ is substituted.
- 6) The condition is checked. If it is fulfilled the calculations stop, if not, improved value of $\lambda \rightarrow \lambda^{(1)}$ is calculated using equation.
- 7) Calculations of the improved values of $P_i \rightarrow P_i^{(1)}$ is done as using equations and so on. Iterations continue until condition is satisfied.
- 8) The optimal solution is used to calculate the total cost, using equation (5).

VI SIMULATION RESULTS

In this paper used IEEE 30 bus system data with six generating units and solve the EED problem using Min-Max and Max-Max price penalty factor. The deferent power demand values are considered. $P_D = 500; 600; 700; 800; 900; 1000$ [MW]. The EED problem is simulated in MATLAB 2013a version of 3GB RAM. In the table 1. IEEE 30 bus system data will be given with six generating units. In this table 1 the generators limits, fuel cost coefficients and emissions coefficients values are given.

In the table 2 the simulation result will be shown. In these table show the lambda values, power demand in [MW], the real power of the generator in [MW], fuel cost in [\$/hr], emissions values in [kg/hr], and CEED fuel cost in [\$/hr] for the power demand is 500 [MW] to 1000 [MW]. In the EED problem uses the Min-Max price penalty factor h_i given by the equation (6) in order to defined the optimal solution using Lagrange's method. The lambda values is assumed as 8. The

maximum number of iterations considered is 2000. In table 2 the single area power dispatch problem is solved by the equation (5) and is used to Min-Max price penalty factor. These price penalty factor equation is written in short form as

$$F_T = F_C + h_i (E_T) \quad [$/hr]$$

$$\text{Where } h_i = \frac{F_c (P_{i,\min})}{E_c (P_{i,\max})}, \quad i=1, n \quad [$/hr],$$

$$F_T = F_C + h_i (E_T) \quad [$/hr],$$

In table 3 the single area power dispatch problem is solved by the equation (5) and using the Max-Max price penalty factor to solved by the equation (7), in short form it is written as

$$\text{Where } h_i = \frac{F_c (P_{i,\max})}{E_c (P_{i,\max})}, \quad i=1, n \quad [$/hr],$$

Figure 1 shows the one line diagram of IEEE 30 bus system

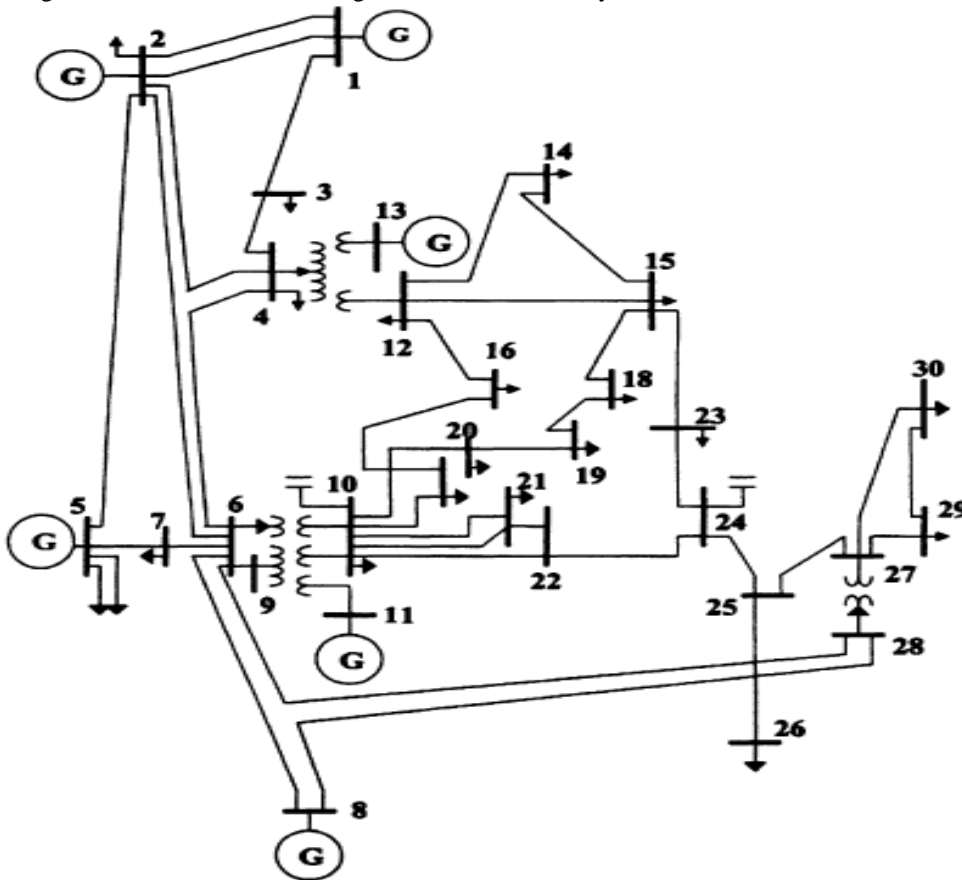


Figure1:IEEE30bus system

In figure 2 show the fuel cost values in [\$/hr] for the single area dispatch problem using Min-Max and Max-Max price penalty factors.

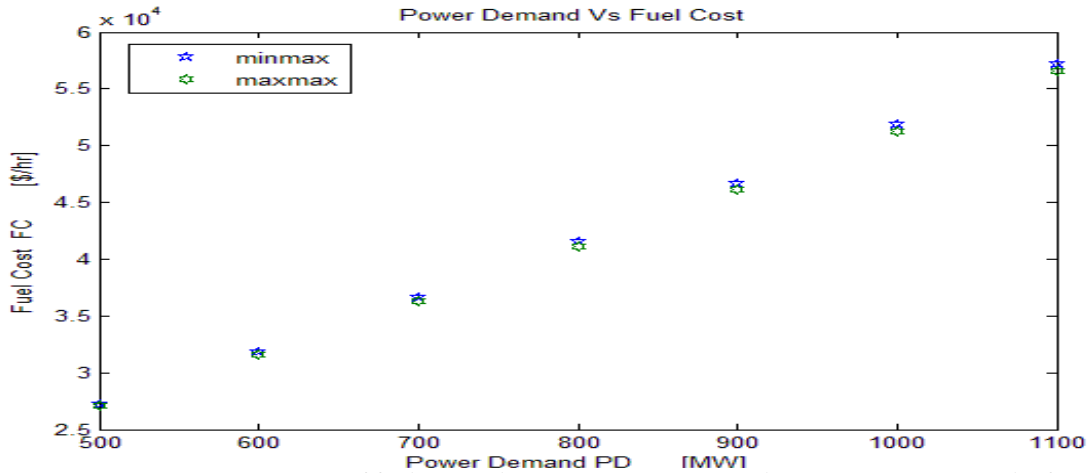


Figure 2 comparison of fuel cost values using Min-Max and Max-Max price penalty factors

In these figure the upper point describe the fuel cost values in [\$/hr] when using Max-Max price penalty factor and the bottom point describe the fuel cost values in [\$/hr] when using Min-Max price penalty factor. In that two price penalty factor compare and then the Min-Max price penalty factor has less fuel cost to Max-Max price penalty factor approach for the single area power dispatch problem for getter values the power demand. The fuel cost is the same for the smaller values of power demand.

Figure 3 show the emissions values in [kg/hr] for the single area dispatch problem using Min-Max and Max-Max price penalty factor.

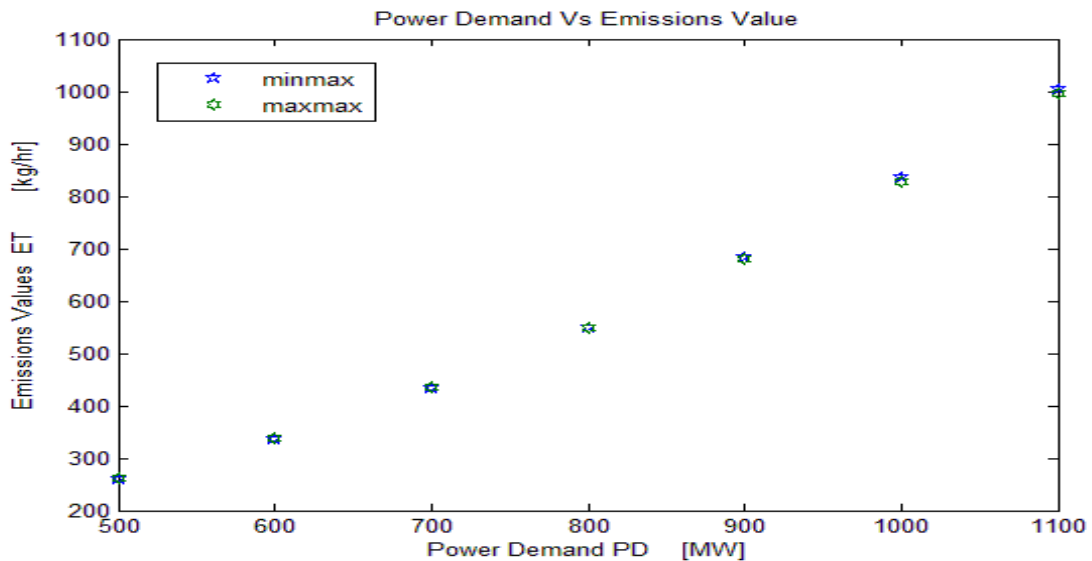


Figure 3 comparison of emission values using Min-Max and Max-Max price penalty factors

In the figure 3 the upper points describe the emissions values in [kg/hr] when using Min-Max price penalty factor. The bottom points describe the emissions in [\$/hr] when using Max-Max price penalty factor. It is concludes that the Min-Max price penalty factor has higher emissions when compared to Max-Max price penalty factor for the single area dispatch problem for bigger values of power demand. The emissions values is the same for the smaller values of the power demands.

In the figure 4 the CEED fuel cost values in [\$/hr] of the single area dispatch problem using Min-Max and Max-Max price penalty factors.

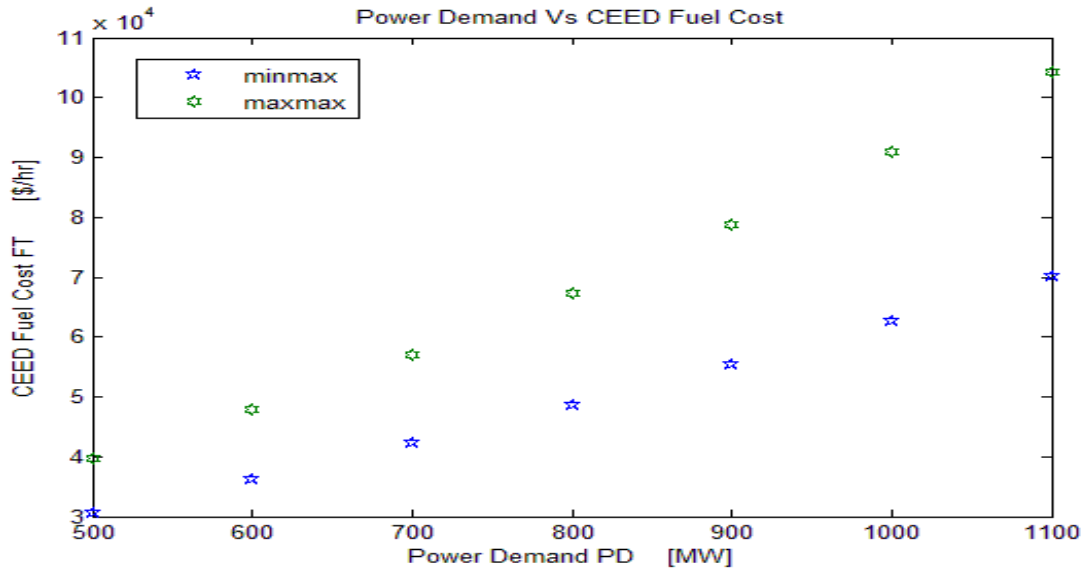


Figure 4 comparison of CEED fuel cost vales using Min-Max and Max-Max price penalty factors

In figure 4 the upper point describe the CEED fuel cost values in [\$/hr] when using Max-Max price penalty factor. The bottom point describe the CEED fuel cost values in [\$/hr] when using Min-Max price penalty factor. It is conclude that the result of the Min-Max and Max-Max price penalty factor for the single area the Max-Max price penalty factor is higher CEED fuel cost when compare of the Min-Max price penalty factor.

Table 4 describe the comparison of the simulation result of single area dispatch problem using Min-Max and Max-Max price penalty factor. The compassion is done for the maximum power demand $P_D = 1000$ [MW] because the highest difference in the values of the criteria appeared for the maximum power demand. The values of the criteria for the case of Min-Max penalty factor are taken for the basis and the difference for the Max-Max penalty factor is given in percentage.

Table 4. Comparison of simulation results using Min-Max and Max-Max price penalty factors

Criterion	Min-Max price penalty factor	Max-Max price penalty factor
Fuel cost F_c [\$/hr]	100 %	101.17 %
Combined Economic emission dispatch fuel cost F_T [\$/hr]	100 %	150.55 %
Emission value E_T [kg/hr]	100 %	95.97 %

VII. CONCLUSION

The Lagrange's method and algorithm for solution of IEEE 30 bus system of single area dispatch problem using Min-Max and Max-Max price penalty factors is developed. The simulated results show that Min-Max price penalty factor provides better optimization solution for the single area dispatch problem in comparison with Max-Max price penalty factor. The simulation results show that Min-Max price penalty factor yields a minimum overall cost for single area power system dispatch problem. It can be concluded that the minimum overall cost for the single area dispatch problem can be provided when using Min-Max Price penalty factor in comparison to Max-Max price penalty factor.

The results also show that the power loss and emission values are less in Max-Max price penalty factor when compared to Min-Max price penalty factor. The CEED fuel cost values are less with Min-Max price penalty factor by 56.90% in comparison to the solution using Max-Max price penalty factor. It concludes that the Min-Max price

penalty factor attains the minimum CEED fuel cost at demand value as received when using Lagrange’s method shown figure in linechart.

Generator Numbers	Generator limits[MW]		Fuelcost coefficients			Emission coefficients		
	P_{max}	P_{min}	a	b	c	d	e	f
1	125	10	0.1525	38.540	756.800	0.00420	0.3300	13.860
2	150	10	0.1060	46.160	451.325	0.00420	0.3300	13.860
3	225	35	0.0280	40.400	1050.000	0.00683	-0.5455	40.267
4	210	35	0.0355	38.310	1243.530	0.00683	-0.5455	40.267
5	325	130	0.0211	36.328	1658.570	0.00460	-0.5112	42.900
6	315	125	0.0180	38.270	1356.660	0.00460	-0.5112	42.900

Table 1: IEEE 30Bus System Data

PD [MW]	Lambda	P1 [MW]	P2 [MW]	P3 [MW]	P4 [MW]	P5 [MW]	P6 [MW]	FC [\$/hr]	ET [Kg/hr]	FT [\$/hr]
500	55.851	44.048	35.849	86.186	78.916	130.000	125.000	27380	256.431	30762
600	62.035	60.085	59.525		105.893	130.000	125.000	32333	333.994	36430
700	66.592	71.907	76.978	144.052	125.779	144.393	136.890	37301	430.693	42479
800	70.497	82.034	91.929	165.086	142.813	162.711	155.426	42324	542.979	48855
900	74.400	92.161	106.879	186.120	159.848	181.029	173.962	47499	674.260	55622
1000	78.305	102.288	121.830	207.154	176.883	199.347	192.498	52824	824.535	62779

Table2: Solution of Multi-Criteria Power System Dispatch Problem using Min-Max Price Penalty factor

PD [MW]	Lambda	P1 [MW]	P2 [MW]	P3 [MW]	P4 [MW]	P5 [MW]	P6 [MW]	FC [\$/hr]	ET [Kg/hr]	FT [\$/hr]
500	77.459	19.999	14.899	93.051	90.064	143.648	138.339	27092.5	261.634	39623.00
600	87.526	31.723	28.662	108.405	103.960	166.507	160.744	31628.6	338.992	47872.30
700	97.592	43.446	42.426	123.758	117.855	189.366	183.148	36313.9	434.380	57128.21
800	107.658	55.170	56.189	139.112	131.751	212.225	205.552	41148.3	547.796	67390.72
900	117.724	66.894	69.952	154.466	145.647	235.084	227.957	46131.8	679.241	78659.84
1000	127.790	78.617	83.716	169.820	159.542	257.944	250.361	51264.5	828.715	90935.56

Table3: Solution of single area Power System Dispatch Problem using Max-Max Price Penalty factor

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