

**ELLIPTICAL FOURIER DESCRIPTOR FOR SHAPE BASED IMAGE
CLASSIFICATION**Chetan Javiya¹, Bijal Talati²¹Computer Engineering, SVIT vasad,²Computer Engineering, SVIT vasad,

Abstract — *The amount of images being uploaded to the internet is rapidly increasing. Therefore, an important research topic is how to manage and perform classification of a large amount of images by using the computational resource in an efficient manner. For that purpose, Classification of the images should be based on contents of images. Various image processing methods can be used to classify the images such as color based, shape based, texture based, histogram based etc. In this paper, we have used shape based Elliptical Fourier descriptors for content based image classification. So, efficient algorithm should be designed for classification of images with proper parameters like number of harmonics, sampling points and parameter for classification of images.*

Keywords- *image classification, shape based descriptors, efficiency, elliptical fourier analysis, Elliptical fourier descriptor.*

I. INTRODUCTION

As we know, every minute there are millions of images uploaded to internet. So, managing and analyzing image data is crucial task which gives opportunity for research. Traditionally, images were classified using textual information as the name of the image. The benefit from textual explanation of image is that it can provide user with key word which helps to understand the information contained in image. But these traditional methods like textual explanation cannot be useful for visual/audio data. The specified disadvantages occurs because: (i) A standard description language is not followed by audio/visual data, (ii) They are inconsistent, (iii) They are subjective, (iv) Time consuming.

As a result, there has been a new focus on developing image classification techniques which have following properties: (i) They should have the capability to classify image based on their contents; (ii) They should be domain independent; and (iii) They should be automated.

Automated classification of similar images based on its content is a challenging task since it deals with pictorial content and the results are not always Accurate. During classification, normally images that are not in same class are also classified as same class. Efficient classification algorithm is crucial to make with variety of images. The Classification algorithm uses the visual contents of an image such as *shape, texture, color, faces, spatial layout*, etc. in order to represent and classification of the images [1].

This work will focus on the designing shape based image classification algorithm using Elliptical fourier descriptor, which gives efficient classification algorithm. There are several advantages to represent a closed edge of object with a set of Elliptical Fourier Descriptors (EFD): invariance to translation, rotation and scaling. Next section describes the elliptical fourier descriptor. In second section analysis of EFD has been done. Third section shows the results generated using EFD. Fourth section gives information about future work and conclusion.

II. ELLIPTICAL FOURIER DESCRIPTOR

shape can be modeled using normalized Elliptic Fourier descriptors of a closed-contour of the edge (Kuhl and Giardina, 1982). The closed-contour can be defined with differential chain code, represented as a sequence of vectors of unit length and directional coding as shown by an example in Fig. 1[2].

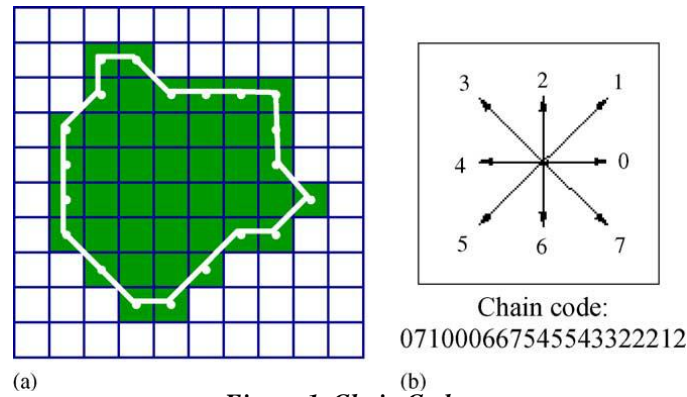


Figure 1. Chain Codes

Pixels were used to describe the chain code, starting from the upper left pixel of the contour and tracing the boundary clockwise [3]. The chain code enumeration is completed when the original starting point is reached. Using the chain code, a truncated Fourier series expansion of the closed-contour which is a projection on the x and y axes is obtained, given as:

$$x_N(t) = A_0 + \sum_{n=1}^N a_n \cos\left(\frac{2n\pi t}{T}\right) + b_n \sin\left(\frac{2n\pi t}{T}\right)$$

$$y_N(t) = C_0 + \sum_{n=1}^N c_n \cos\left(\frac{2n\pi t}{T}\right) + d_n \sin\left(\frac{2n\pi t}{T}\right)$$

where t is the step required to move 1 pixel along the closed-contour, such that $t_{p-1} < t < t_p$ for code values of p , within the range of $1 < p < k$ and k gives the total number of codes describing the contour of boundary. n shows the number of Fourier harmonics required to generate the approximation of the boundary (each harmonic has four coefficients). here, T is the basic period of the chain code of contour, or steps needed to traverse the entire contour, $T = t_k$. A_0 and C_0 are the bias coefficients, corresponding to a frequency of 0. These coefficients (A_0 and C_0) are related to image translation and N is the total number of EF harmonics needed to generate an accurate approximation of the boundary.

For each harmonic, the n^{th} set of four harmonic coefficients a_n , b_n , c_n and d_n was defined as:

$$a_n = \frac{T}{2n^2\pi^2} \sum_{p=1}^K \frac{\Delta x_p}{\Delta t_p} \left[\cos\left(\frac{2n\pi t_p}{T}\right) - \cos\left(\frac{2n\pi t_{p-1}}{T}\right) \right],$$

$$b_n = \frac{T}{2n^2\pi^2} \sum_{p=1}^K \frac{\Delta x_p}{\Delta t_p} \left[\sin\left(\frac{2n\pi t_p}{T}\right) - \sin\left(\frac{2n\pi t_{p-1}}{T}\right) \right],$$

$$c_n = \frac{T}{2n^2\pi^2} \sum_{p=1}^K \frac{\Delta y_p}{\Delta t_p} \left[\cos\left(\frac{2n\pi t_p}{T}\right) - \cos\left(\frac{2n\pi t_{p-1}}{T}\right) \right],$$

$$d_n = \frac{T}{2n^2\pi^2} \sum_{p=1}^K \frac{\Delta y_p}{\Delta t_p} \left[\sin\left(\frac{2n\pi t_p}{T}\right) - \sin\left(\frac{2n\pi t_{p-1}}{T}\right) \right],$$

Where t_p shows the number of steps required to traverse the first p components or links of the chain code generated from contour. x_p and y_p are the spatial changes in the x and y directions of the chain code, respectively, at link p . These values may be 1, 0 or -1 depending on the orientation of link p . t_p is the step change required to traverse link p of the chain code.

Shape descriptions should be invariant with rotation, size, translation and starting point on the contour, if they are going to be useful for any size of objects in image. These are also important if the shape is used as a template for texture analysis. Object venation and texture will be also subject to object's orientation, size and scan direction. Rotation factor, which is defined as the angle between the starting point and the first semi-major axis or the same counterclockwise rotation about the contour. defined as:

$$\theta_1 = \frac{1}{2} \arctan \left[\frac{2(a_1 b_1 + c_1 d_1)}{a_1^2 + c_1^2 - b_1^2 - d_1^2} \right] \quad 0 \leq \theta_1 \leq 2\pi$$

Where a_1, b_1, c_1 and d_1 are the first set of harmonic coefficients.

A rotational transformation matrix using radians was then applied to all of the harmonic coefficients to provide a new set for the standard object orientation, which is given as:

$$\begin{bmatrix} a_n^* & c_n^* \\ b_n^* & d_n^* \end{bmatrix} = \begin{bmatrix} \cos \theta_1 & \sin \theta_1 \\ -\sin \theta_1 & \cos \theta_1 \end{bmatrix} \begin{bmatrix} a_n & c_n \\ b_n & d_n \end{bmatrix}$$

After a starting point correction, a rotation invariant normalization operation was next performed. The semi-major axis of the first harmonic was rotated by angle Θ until it was parallel to the positive x -axis of the first quadrant. The rotation angle ψ_1 was defined using the elliptic locus coordinates of the first point, given as:

$$x_1^*(t^*) = a_1^* \cos \left(\frac{2\pi t^*}{T} \right) + b_1^* \sin \left(\frac{2\pi t^*}{T} \right),$$

$$y_1^*(t^*) = c_1^* \cos \left(\frac{2\pi t^*}{T} \right) + d_1^* \sin \left(\frac{2\pi t^*}{T} \right)$$

Considering that the first harmonic phasor can be aligned with the semi-major axis at the first point, $t^* = 0$, the rotational angle ψ_1 can be obtained as:

$$\psi_1 = \arctan \left[\frac{y_1^*(0)}{x_1^*(0)} \right] = \arctan \left(\frac{c_1^*}{a_1^*} \right), \quad \text{where } 0 \leq \psi_1 < 2\pi$$

Size invariance was accomplished by dividing each coefficient by the magnitude E^* of the semi-major axis, defined as:

$$E^*(0) = (x_1^*(0)^2 + y_1^*(0)^2)^{1/2} = (a_1^{*2} + c_1^{*2})^{1/2}$$

Setting the bias term A_0 and C_0 to zero made the EF coefficient set invariant to translation. The final normalized and rotated EF boundary approximation for the closed-contour of the shape is shown in Fig.2.

III. ANALYSIS AND RESULTS

Classification and Elliptic Fourier processing was performed, using MATLAB script (Version 6.1 with functions from the Image Processing Toolbox (The MathWorks, Inc., 2000). All computations were all performed using WINDOWS 8 Professional® on a Dell 3.1 GHz, intel core2duo computer with two gigabyte of memory.

A data set that has already been used for the evaluation of different Fourier descriptors in the study is part of the MPEG-7 CE-Shape-1 database. It consists of 1,400 shapes that have been classified into 70 classes with 20 similar items in each class. Figure 2 shows sample shapes from this data set. As pointed out by the authors of the data set, a 100% retrieval rate is impossible because some shapes are more similar to the shapes from different classes than to their own class so that 'it is not possible to group them into the same class using only shape knowledge. In some images, there are noise pixels which form additional small random shapes. In order to ignore this noise, we have computed the contour of the largest connected component for each image only. Our classification is based on the coefficients of fourier, and each quadruple (a_i, b_i, c_i, d_i) denotes an elliptical harmonic of a closed contour. Moreover, each quadruple will be modified to be invariant from starting point, translation, rotation and scaling.

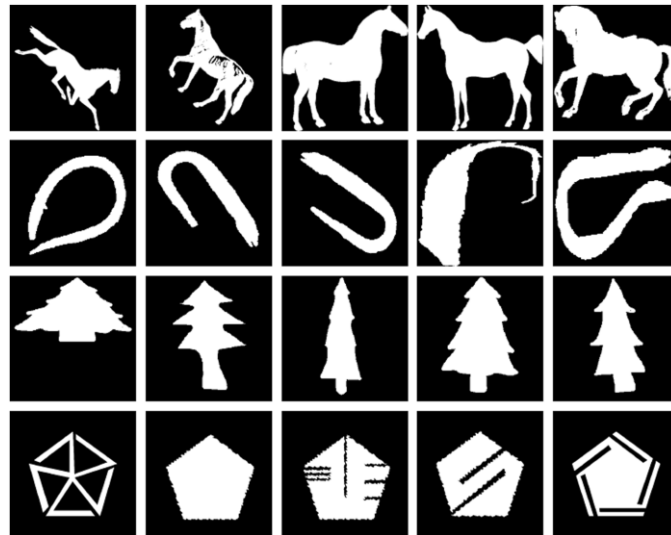


Figure 2. mpeg 7 shapes

A) Selection of Number of Harmonics

The number of sets of EF coefficients increases as the number of harmonics increases. For each harmonic set, four coefficients are always generated. For discrimination proposes, all the fourier coefficients can be considered. The total number of coefficients was computed to be:

$$N_h = 4h$$

where h is harmonic number. Harmonic number should be selected such that there should be balance between accuracy and computational efficiency. there is tread off between accuracy and efficiency when we are dealing with number of harmonic selection. Here are some images of same object with different number of harmonics in Figure 3.

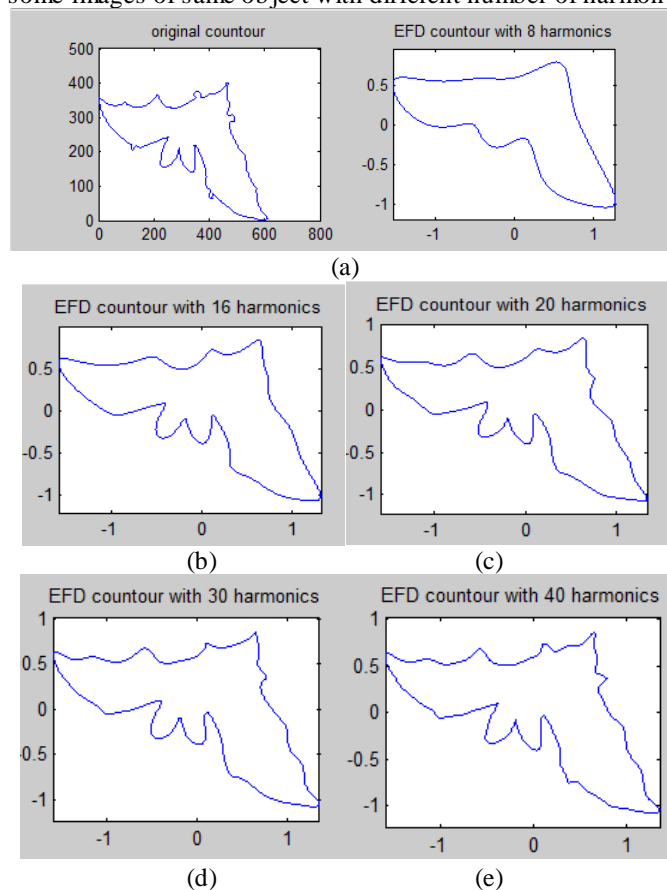


Figure 3. EFDs with harmonics

Fig. 3 (a) shows the original contour plot of image and its EFD contour with harmonic number 8. It is observed that boundary has become smoother. So, noise near boundary is removed automatically. Which is also one of the benefits using EFD. In Fig. 3 (b) and (c), EFD contour plots with 16 and 20 number of harmonics are shown respectively. We can observe that there are more steepness of curve in image using 20 number of harmonics compared to 16 number of harmonics. And shape is represented properly using 20 numbers of harmonics. In Fig 3 (d) and (e), there is no more difference between EFD contour plots of image using 20, 30, and 40 number of harmonics as compared to their computational efficiency. So, we had selected 20 harmonics for classification of images of each image for their similarity measure. Where, balance between computational efficiency and accuracy has been achieved. Figure 4 shows some contour plots of images using 20 number of harmonics.

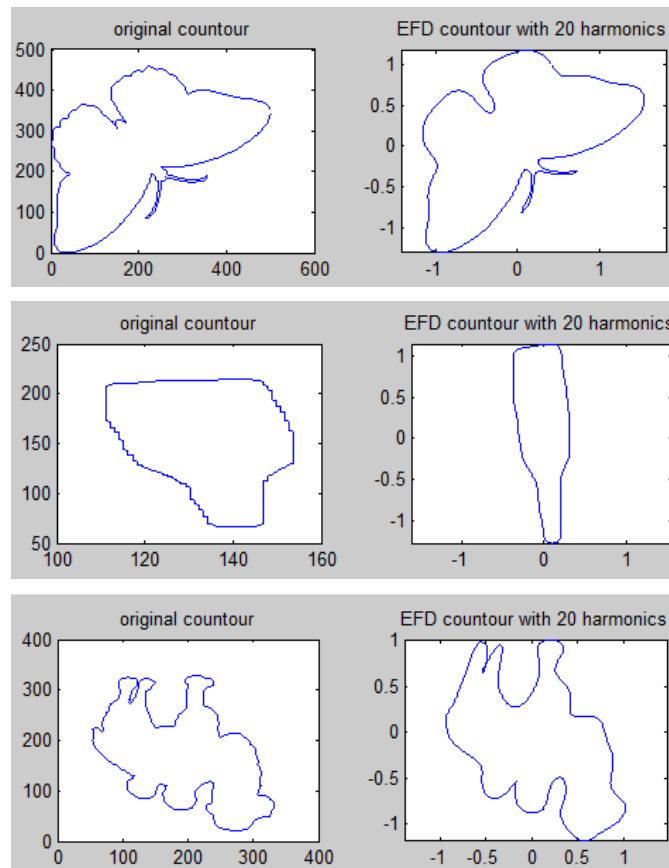


Figure 4. EFDs with 20 no. of harmonics

B) EFD analysis for similarity measure

We had done manual analysis of the EFD values using 20 harmonics of five images of the same category. We were expecting the similarity between same coefficients of different images of same category. So, we had manually analyzed the 'a' coefficient of different images of category apple. Where, we had calculated the different functions on descriptor values shown in table 1.

Sr.no.	20 coefficient approximation				
	Apple				
	a ₁	a ₂	a ₃	a ₄	a ₅
1	-1.00	-1.00	-1.00	1.00	-1.00
2	-0.14	-0.12	-0.24	0.20	-0.22
3	0.01	0.02	-0.02	0.04	-0.05
4	0.01	-0.01	0.01	-0.06	0.04
5	-0.02	0.00	0.02	0.03	0.01

6	0.01	-0.01	-0.03	-0.02	-0.03
7	-0.02	0.01	0.02	0.01	0.02
8	0.02	0.00	-0.02	-0.01	-0.02
9	-0.02	0.00	0.00	0.02	0.00
10	0.01	0.00	0.00	-0.01	0.00
11	0.00	0.00	-0.01	0.01	-0.01
12	0.00	0.00	0.01	-0.01	-0.01
13	0.00	0.00	0.00	0.01	0.01
14	0.00	0.01	0.00	-0.01	-0.01
15	0.00	-0.01	0.00	0.00	0.00
16	0.00	0.01	0.00	0.01	0.00
17	0.00	0.00	0.00	-0.01	0.00
18	0.00	0.00	0.00	0.00	0.00
19	0.00	0.00	0.00	0.00	0.00
20	0.00	0.00	0.00	0.00	0.01
sum	-0.13	-0.11	-0.25	0.20	-0.24
Average	-0.01	-0.01	-0.01	0.01	-0.01
min	-0.14	-0.12	-0.24	-0.06	-0.22
max	0.02	0.02	0.02	0.20	0.04

Table 1.

As we you can observe there are no similarities between values of different images using same function. So, here we cannot use a simple threshold value using any mathematical functions used in table 1 for classification of images. EFD Values of same function of different images varies too much. So, we had diverted our research to find the differences between images for searching a parameter value to differentiate images.

C) Classification and similarity measure

As we have seen that, EFD coefficient's functions cannot differentiate the images. We tried to measure dissimilarity between images using subtraction of EFD coefficient's values of images of same categories as well as other categories. Results show that it could be used for similarity measure. But as we know EFD's values are negative as well as positive. So, some mathematical function should be used for similarity measure which gives difference in images of different category, which should have large difference between same category and other category.

We had tried different similarity measure and one of them we had selected is Euclidian distance measure, given by:

$$SM = \sum_{i,j=1}^{i,j=20} \sqrt{(a_i - a_j)^2} + \sqrt{(b_i - b_j)^2} + \sqrt{(c_i - c_j)^2} + \sqrt{(d_i - d_j)^2}$$

Where a_i, b_i, c_i, d_i are EFD coefficients of image of known class and a_j, b_j, c_j, d_j are EFD coefficients of test image to be classified. Euclidian Distance between test image and each image of all categories of images are computed and image is classified based on minimum Euclidian distance between images.

Now, let us analyze the effect of rotation invariance in EFD. We have computed Euclidian distance between images of same category without rotation invariance in Table 2(a). We have also computed the Euclidian distance between images of different category without rotation invariance but with translation and scale invariance shown in table 2(b).

Sr.no	without rotation invariance				
	Applle1-Apple2				
	ai-aj	bi-bj	ci-cj	di-dj	
1	0.15	0.06	0.08	0.17	
2	0.01	0.15	0.04	0.02	
3	0.02	0.05	0.00	0.03	
4	0.04	0.02	0.01	0.03	
5	0.06	0.01	0.00	0.02	
6	0.07	0.02	0.00	0.02	
7	0.05	0.04	0.01	0.00	
8	0.02	0.04	0.01	0.00	
9	0.00	0.03	0.02	0.01	
10	0.00	0.00	0.01	0.01	
11	0.01	0.00	0.00	0.02	
12	0.02	0.00	0.01	0.01	
13	0.01	0.01	0.02	0.01	
14	0.00	0.01	0.01	0.00	
15	0.01	0.01	0.01	0.01	
16	0.01	0.00	0.00	0.01	
17	0.01	0.01	0.00	0.01	
18	0.00	0.01	0.00	0.00	
19	0.00	0.00	0.00	0.00	
20	0.00	0.00	0.00	0.00	
sum	0.48	0.48	0.24	0.38	1.58
Avg	0.02	0.02	0.01	0.02	0.39
Min	0.15	0.15	0.08	0.17	0.17
max	0.00	0.00	0.00	0.00	0.00

Table 2(a).

	without rotation invariance				
	apple1-apple3				
	ai-aj	bi-bj	ci-cj	di-dj	
Sum	1.34	0.76	0.65	0.68	2.24
Avg	0.08	0.05	0.04	0.04	0.56
Min	0.89	0.54	0.41	0.41	0.27
Max	0.00	0.00	0.00	0.00	0.00

Table 2(b).

We can observe that there is too much difference in distance values calculated for same category of images in table 2(a) and (b). (apple1-apple2 = 1,58 , apple1-apple3 = 2.24).

So, it is observed that we can not classify images without rotation invariance of objects in images Here, there are some manual analysis results with rotation, translation and scale invariance and Euclidian distance shown in table 3.

size,rotation and translation invariance					
apple1-apple2					
	Dista	Distb	Distc	Distd	
sum:	0.242358	0.323610	0.300467	0.484086	1.350521
avg	0.012118	0.016180	0.015023	0.024204	0.016882
max	0.030221	0.048469	0.059016	0.145549	0.145549
min	0.000000	0.001150	0.000000	0.000323	0.000000

Table 3(a).

size,rotation and translation invariance					
apple1-apple3					
	Dista	Distb	Distc	Distd	
sum:	0.354915	0.358413	0.616608	0.453403	1.783339
avg	0.017746	0.017921	0.030830	0.022670	0.022292
max	0.103085	0.057521	0.124902	0.127258	0.127258
min	0.000000	0.000466	0.000000	0.000342	0.000000

Table 3(b).

size,rotation and translation invariance					
apple1-apple5					
	Dista	Distb	Distc	Distd	
sum:	0.370186	0.251077	0.459385	0.698750	1.779397
avg	0.018509	0.012554	0.022969	0.034938	0.022242
max	0.082367	0.066069	0.087074	0.131146	0.131146
min	0.000000	0.000651	0.000000	0.000089	0

Table 3(c).

size, rotation and translation invariance					
apple1-bat1					
	Dista	Distb	Distc	Distd	
sum	2.409573	0.723776	0.293219	0.515600	3.942167
avg	0.120479	0.036189	0.014661	0.025780	0.049277
max	2.000000	0.324680	0.075025	0.123180	2.000000
min	0.000876	0.000376	0.000000	0.000439	0.000000

Table 3(d).

size, rotation and translation invariance					
apple1-beetle1					
	Dista	Distb	Distc	Distd	
sum	0.489175	1.061380	0.989506	1.202238	3.742299
avg	0.024459	0.053069	0.049475	0.060112	0.046779
max	0.089607	0.279146	0.216145	0.318306	0.089607
min	0.000000	0.000399	0.000000	0.000383	0.000000

Table 3(e).

	size,rotation and translation invariance				
	apple1-bell1				
	Dista	Distb	Distc	Distd	
sum	2.218507	0.442899	0.347244	0.397786	3.406436
avg	0.110925	0.022145	0.017362	0.019889	0.04258
max	2.000000	0.169437	0.094189	0.086692	2
min	0.000388	0.001455	0.000000	0.000276	0

Table 3(f).

Table 3 (a), (b), (c) contains different mathematical functions applied on Euclidian distances of all coefficients of the images of same category. Sum of sum of all coefficients (a_i, b_i, c_i, d_i) is taken as measure for classification of images. As, we can observe that difference of values between same category are nearer to each other and which is also the minimum amongst all other category of images (i.e. ~ 1.5 in Table 3 (a), (b), (c)).

Table 3 (d), (e), (f) shows distance measure values of different category images. Where distance measure gives the values which are too far from values observed between same categories.(i.e ~ 3.6 in Table 3 (d), (e), (f)). So, manual analysis done above gives the surety that the parameter selected for classification of images would give the good results. So, FED with Euclidian distance between images could be used for image classification.

IV. CONCLUSION

EFD preserves the shape information for accurate classification and removes the noise present near boundary of an Object in image. 'a' coefficient stores more information of shape compare to other coefficients. As we use more number of harmonics accuracy increases and computation cost also increases. So, balancing the computation cost and accuracy requires proper number of harmonics in EFD. For that we have chosen 20 number of harmonics for our dataset. It can also classify an object which has different projection but same boundary feature because it encapsulate the information contain in boundary. Euclidian distances between coefficients of images of same category are minimum compared to other categories. We had chosen 'sum of sum' function for differentiating classes as it gives good similarity measure.

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